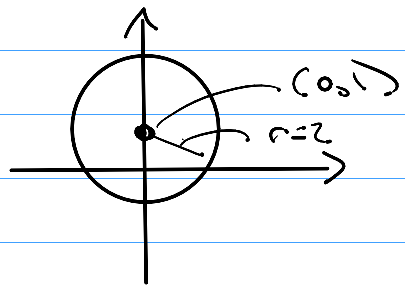
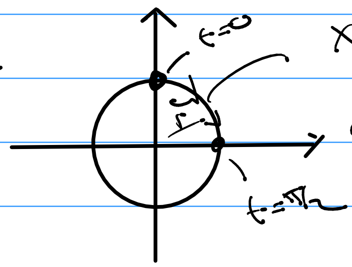
Math 243

Q's #33 $x^2 + (y-1)^2 = (2)^2$



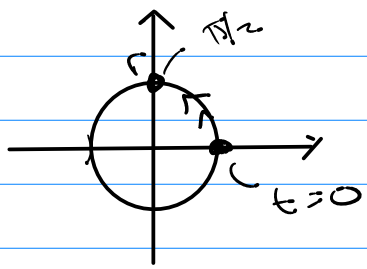
$(x(t), y(t))$? $\alpha \le t \le \beta$

circles:



$x(t) = r\sin t$ $0 \le t \le \pi/2$

$y(t) = r\cos t$

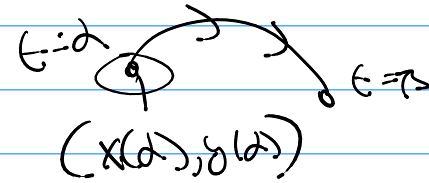


$x(t) = r\cos t$ $0 \le t \le \pi/2$

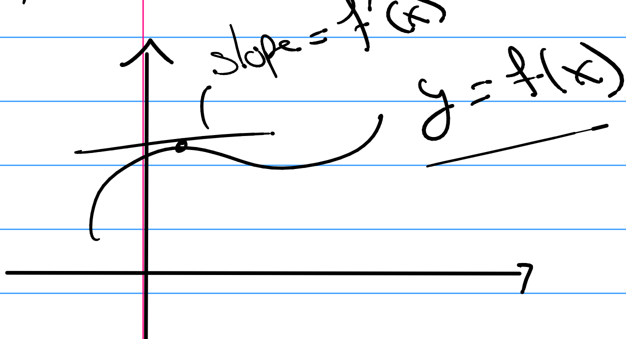
$y(t) = r\sin t$

a) $x(t) = 2\sin t$ $0 \le t \le 2\pi$

$y(t) = 2\cos t + 1$

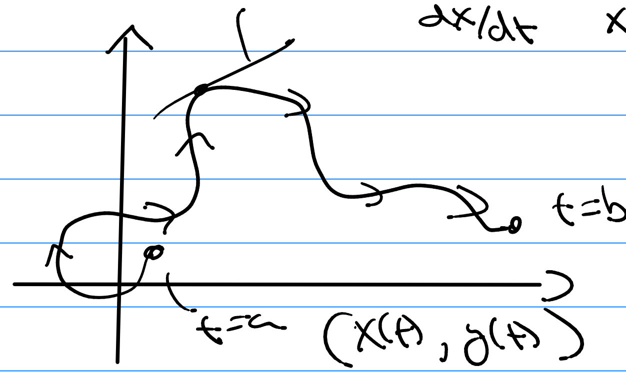


10.2 Parametric Eq's ⟹ Calculus



Now

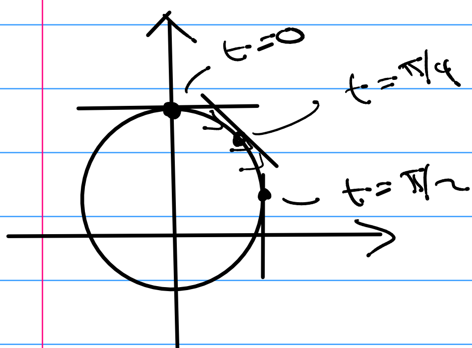
slope = $\dfrac{dy/dt}{dx/dt} = \dfrac{y'}{x'}$



ex $x(t) = 3\sin(t)$ (traces a circle)

$y(t) = 3\cos(t) + 1$

slope $= \boxed{\frac{dy}{dx}} = \frac{dy/dt}{dx/dt} = \frac{-3\sin(t)}{3\cos(t)} = -\tan(t)$



$t = \pi/2$

Note: $y = f(x)$

and $\frac{dy}{dx} = \frac{d}{dx}[\overset{y}{\underset{\text{function}}{f(x)}}] = \frac{dy/dt}{dx/dt} = \frac{\frac{d}{dt}[\overset{\text{function}}{y}]}{dx/dt}$

So $\boxed{\frac{d}{dx}[\square] = \frac{\frac{d}{dt}[\square]}{dx/dt}}$ how to take a derivative of parametric eq'n's

and $f'' = \frac{d}{dx}[f']$

$f''' = \frac{d}{dx}[f'']$

etc.

all together $\quad (x(t), y(t))$

$$\text{slope} = \frac{dy}{dx} = \frac{dy/dt}{dx/dt} =$$

Note this is an expression for dy/dx, a function of t

curvature

2^{nd} deriv

$$= \frac{d}{dx}\left[\frac{dy}{dx}\right] = \frac{\frac{d\left[dy/dx\right]}{dt}}{dx/dt}$$

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ex $\quad x(t) = 3\sin(t) \qquad y(t) = 3\cos(t)+1$

$$\text{slope} = \frac{dy}{dx} = \frac{-3\sin(t)}{3\cos(t)} = -\tan(t)$$

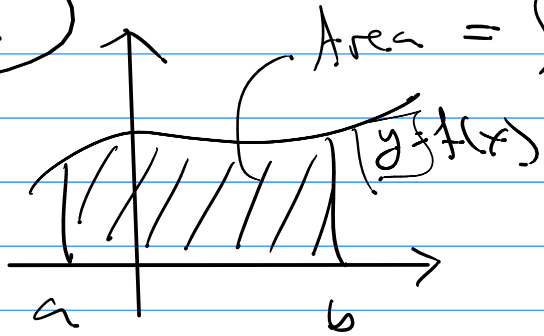
$$2^{nd}\text{ deriv} = \frac{d}{dx}\left[dy/dx\right] = \frac{\frac{d}{dt}\left[-\tan(t)\right]}{dx/dt} = \frac{-\sec^2(t)}{3\cos(t)}$$

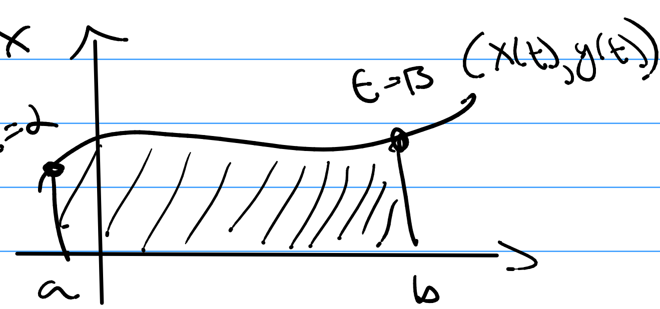
$$2^{nd}\text{ deriv} \quad \frac{d^2y}{dx^2} = -\frac{1}{3}\sec^3(t)$$

$$3^{rd}\text{ deriv.} = \frac{d}{dx}\left[\frac{d^2y}{dx^2}\right] = \frac{\frac{d}{dt}\left[-1/3\,\sec^3(t)\right]}{dx/dt} = \sim \text{finish}$$

Area?

$$\text{Area} = \int_a^b f(x)\,dx$$





$\boxed{\text{parametric}}$

$$A = \int_\alpha^\beta y\, x'\, dt = \boxed{\int_\alpha^\beta y(t)\, x'(t)\, dt}$$

$x = x(t) \qquad y = y(t)$

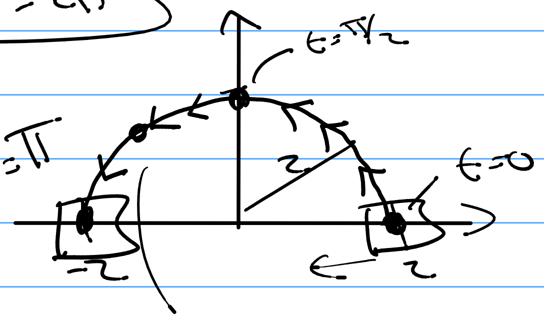
$dx = x'(t)\,dt$

Ex

$$\int_{-2}^{2} \sqrt{4 - x^2}\, dx$$

$$\int_{\pi}^{0} \underbrace{(2\sin(t))}_{y}\underbrace{(-2\sin t)}_{x'}\, dt$$

$\boxed{A = 2\pi}$



$x(t) = 2\cos(t)$

$y(t) = 2\sin(t)$

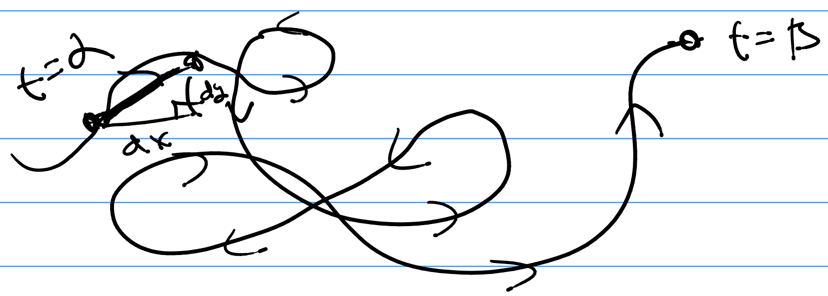
$$-4\int_\pi^0 \sin^2 t\, dt \qquad \boxed{\cos 2t = 1 - 2\sin^2 t}$$

$$= -2\int_\pi^0 (1 - \cos(2t))\,dt$$

$$= -2\left[t - \tfrac{1}{2}\sin(2t)\right]\Big|_\pi^0 = -2\left[0 - \pi\right]$$

$$= \boxed{2\pi}\ ?$$

$\boxed{\text{Arc length}}$



$$L_i = \sqrt{dx^2 + dy^2}$$

$$L_i = \sqrt{dx^2 + dy^2}\cdot\frac{dt}{dt} = \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$$

$$L = \int_{\alpha}^{\beta} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt \qquad x(t),\ y(t) \qquad \frac{dx}{dt} = x',\ \frac{dy}{dt} = y'$$

$$\boxed{L = \int_{\alpha}^{\beta} \sqrt{(x')^2 + (y')^2}\, dt} \qquad y = f(x)$$

$$x(t) = \cos(t) \qquad 0 \le t \le 10$$

$$y(t) = t^2 + 1$$

$$L = \int_0^{10} \sqrt{\sin^2(t) + 4t^2}\, dt = \underline{\text{use approximation.}}$$

$$x(t) = r\cos t \qquad y(t) = r\sin t$$

$$L = \int_0^{2\pi} \sqrt{r^2\sin^2 t + r^2\cos^2 t}\, dt = r\int_0^{2\pi} 1\, dt$$

$$= r\, t \Big|_0^{2\pi} = \boxed{2\pi r}$$
