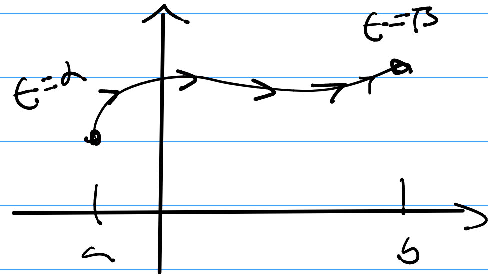
Math 243

Parametric Eq'n's



$y = f(x)$

$\{x(t), y(t)$
$t \in [\alpha, \beta]$

Deriv

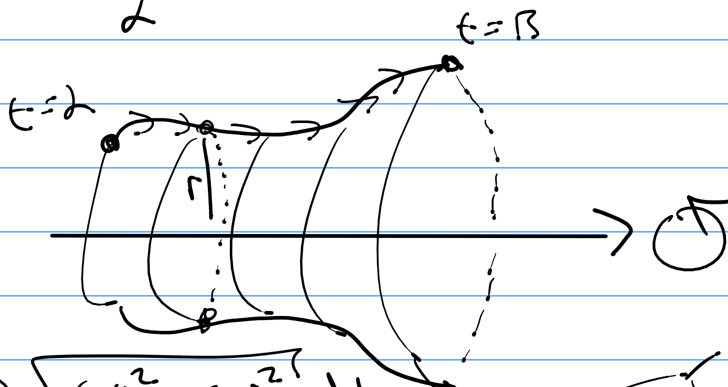
$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} \qquad \frac{d}{dx}[\ ] = \frac{\frac{d}{dt}[\ ]}{dx/dt}$$

Integration

area under f over $[a,b]$ $= \displaystyle\int_\alpha^\beta y(t)\, x'(t)\, dt$

arc length $= \displaystyle\int_\alpha^\beta \sqrt{(x')^2 + (y')^2}\, dt$

Surface Area of rotation



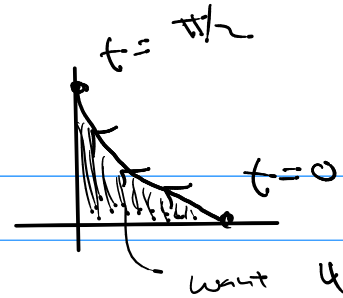
$$SA = \int_\alpha^\beta 2\pi r \sqrt{(x')^2 + (y')^2}\, dt$$

warning in textbook they have $x(t)$ / x or $y(t)$ / y here.

x means rotate about y-axis

y means rotate about x-axis

(ex) $x = \cos^3 t$
$y = \sin^3 t$



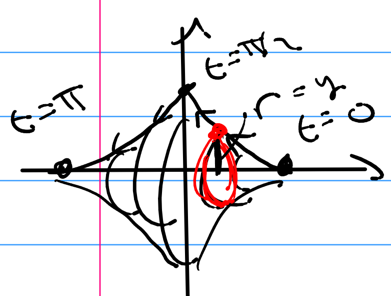
$$\int_\alpha^\beta y(t)\, x'(t)\, dt$$

$$\text{total Area} = 4\int_{\pi/2}^{0} \sin^3(t)\left[3\cos^2(t)\right](-\sin(t))\, dt$$

$$= -12\int_{\pi/2}^{0} \sin^4(t)\cos^2(t)\, dt = 12\int_0^{\pi/2} \sin^4(t)\cos^2(t)\, dt$$

$$= \tfrac{3}{8}\pi$$

S.A. of revolving $x = \cos^3 t$, $y = \sin^3 t$ about x-axis



$$\int_\alpha^\beta 2\pi r\sqrt{(x')^2 + (y')^2}\, dt$$

$$SA = \int_0^\pi 2\pi(\sin^3 t)\sqrt{\left(3\cos^2 t(-\sin t)\right)^2 + \left(3\sin^2 t\cos t\right)^2}\, dt$$

$$SA = \int_0^\pi 2\pi\sin^3 t\sqrt{9\cos^4 t\sin^2 t + 9\sin^4 t\cos^2 t}\, dt$$

$$SA = \int_0^\pi 6\pi\sin^3 t\,|\sin t|\,|\cos t|\sqrt{\cos^2 t + \sin^2 t}\, dt$$

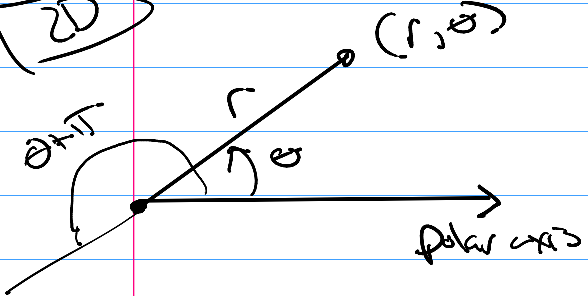
0 to π: $|\sin t| = \sin t$

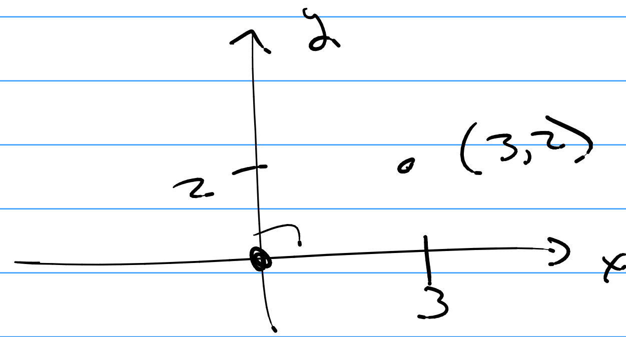
$$SA = \int_0^\pi 6\pi\sin^3(t)(|\sin t|)(|\cos t|)\, dt$$

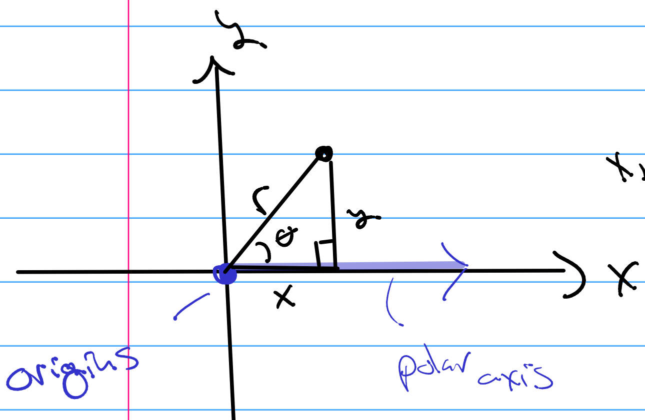
$|\cos t| =$ $\cos t$ for 0 to π/2, $-\cos t$ for π/2 to π

10.3 Polar Coordinates

2D







Coord. conversion eqn's

x, y → $\boxed{\begin{aligned} x^2 + y^2 &= r^2 \\ y/x &= \tan\theta \end{aligned}}$ → r, θ

r?, θ?

r, θ → $\boxed{\begin{aligned} r\cos\theta &= x \\ r\sin\theta &= y \end{aligned}}$ → x, y

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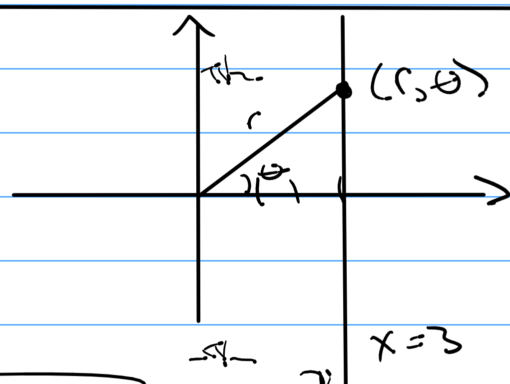
eqn

$x = 3$

$r\cos\theta = 3$

$\boxed{r = 3\sec(\theta)}$



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graph

$r = 2 + 3\sin\theta$

θ	r
0	
π/4	
π/2	
3π/4