Math 243

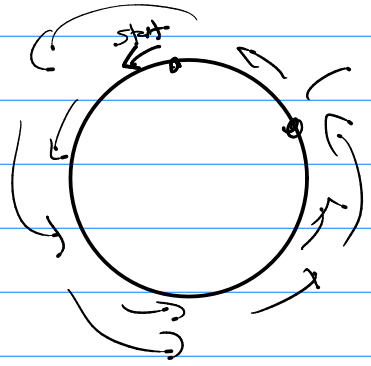
Q's

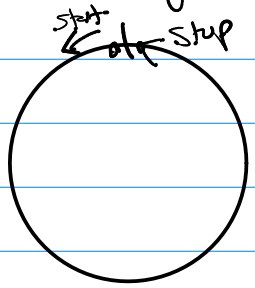
10.2 #51

$x(t) = \sin^2 t$ $\quad t \in [0, 3\pi]$

$y(t) = \cos^2 t$

Distance traveled (vs) Arc length

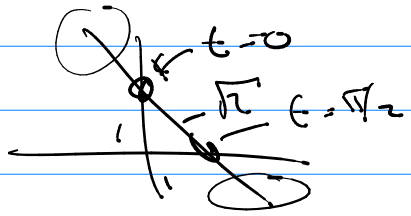




go once.

$$T.D. = \int_0^{3\pi} \sqrt{(x')^2 + (y')^2}\, dt = 6\sqrt{2} \qquad A.L. = \int_0^{\pi/2} \sqrt{(x')^2 + (y')^2}\, dt = \sqrt{2}$$

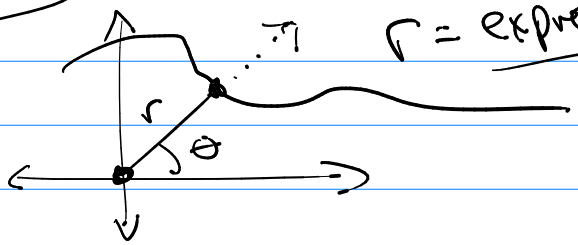
$x = \sin^2 t$ $\quad y = \cos^2 t$



know $\sin^2 t + \cos^2 t = 1$

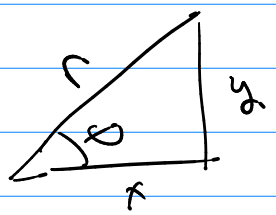
so $x + y = 1 \rightarrow y = -x + 1$

Polar Eqn's



$r =$ expression of θ's

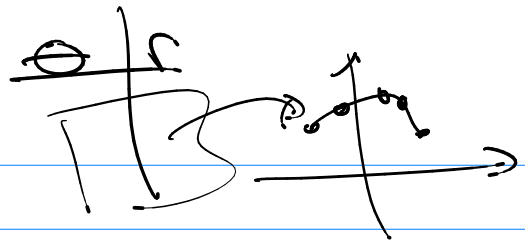
$r = f(\theta)$



$x^2 + y^2 = r^2$ $\qquad y/x = \tan\theta$

$r\cos\theta = x$ $\qquad r\sin\theta = y$

Graphs ① table of values



② Conversion to (x,y)

$r = 5\cos\theta \rightarrow$ x's y's?

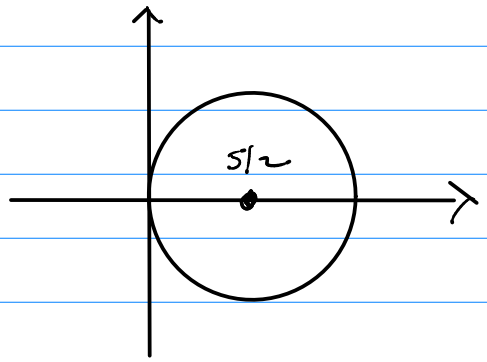
$r = 5\frac{x}{r}$

$r^2 = 5x$

$x^2 + y^2 = 5x$

$x^2 - 5x + (5/2)^2 + y^2 = 0 + (5/2)^2$

$\boxed{(x-5/2)^2 + y^2 = (5/2)^2}$



③ CAS (table of values)

④ geogebra Curve(x(t), y(t), t, start, end)

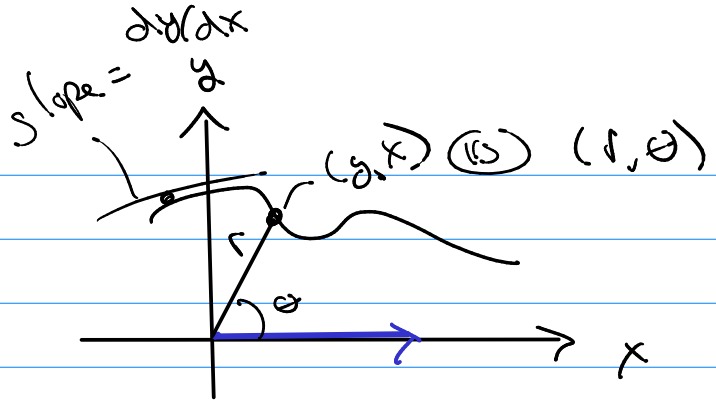
$r = 5\cos\theta$ know: $x = r\cos\theta$ $y = r\sin\theta$

$x(t) = (5\cos(t))\cos(t)$

$y(t) = (5\cos(t))\sin(t)$

Calculus 2

Derivatives: Slope



Need $\frac{dy}{dx}$

① Know $x(t)$, $y(t)$ (parametric)

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$$

② Know: $r = r(\theta)$ (polar function)

$$x(\theta) = r(\theta)\cos(\theta) \qquad y(\theta) = r(\theta)\sin(\theta)$$

↳ parametric eqn's, θ is parameter.

Use ① ②

$$\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{r'(\theta)\sin\theta + r(\theta)\cos\theta}{r'(\theta)\cos\theta - r(\theta)\sin\theta}$$

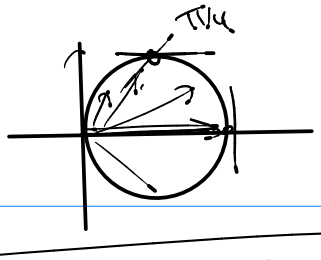
Ex $r = 5\cos\theta$ → parametric eqn $x = 5\cos\theta\cos\theta = 5\cos^2\theta$

$y = 5\cos\theta\sin\theta$

$$\frac{dy}{dx} = \frac{-5\sin\theta\sin\theta + 5\cos\theta\cos\theta}{10\cos\theta\sin\theta} = \frac{-5\sin^2\theta + 5\cos^2\theta}{10\cos\theta\sin\theta}$$

$$\frac{dy}{dx} = \frac{\cos^2\theta - \sin^2\theta}{2\cos\theta\sin\theta} = \frac{\cos 2\theta}{\sin 2\theta} = \cot(2\theta)$$

$$\frac{dy}{dx} = \frac{\cos 2\theta}{\sin 2\theta}$$



Note: polar origin $r = 0$ $r' \neq 0$

$$\frac{dy}{dx} = \tan\theta$$