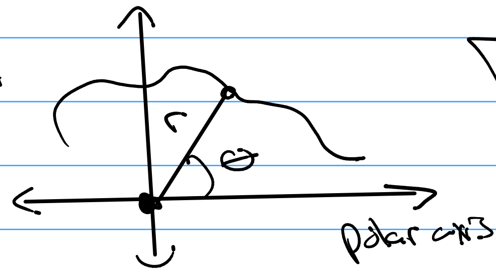
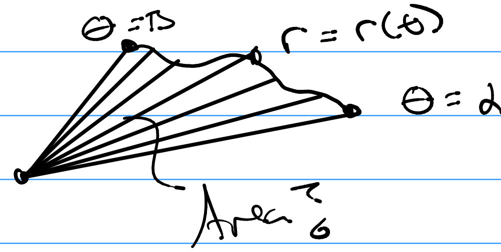
Math 243

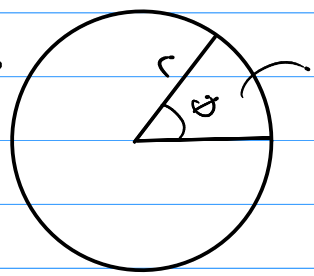
Polar



Q1 Area swept out

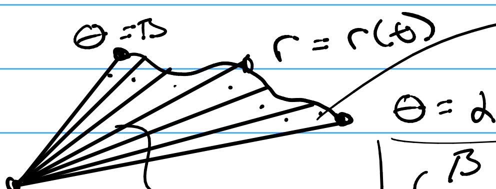


ex

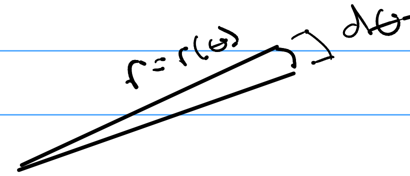


$A = \frac{1}{2}\theta r^2 = \frac{1}{2}r^2\theta$

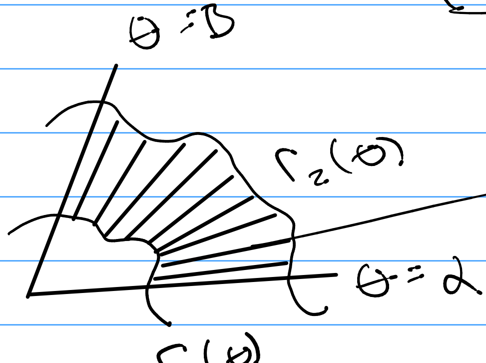
So



small slice

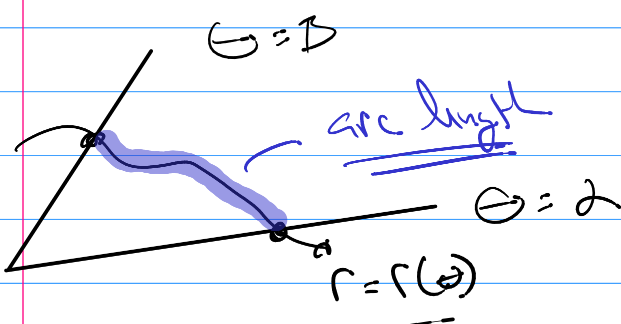


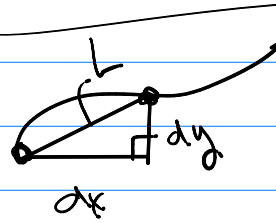
$$\text{Area} = \boxed{\int_\alpha^\beta \frac{1}{2}[r(\theta)]^2\,d\theta}$$



$$\text{area} = \int_\alpha^\beta \frac{1}{2}[r_2]^2\,d\theta - \int_\alpha^\beta \frac{1}{2}[r_1]^2\,d\theta$$

$$\text{area} = \frac{1}{2}\int_\alpha^\beta [r_2]^2 - [r_1]^2\,d\theta$$





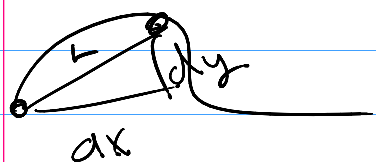
$L = \sqrt{(dx)^2 + (dy)^2}$

used before

problem: $x^?_0$ $y^?_0$

given polar eqn $r = \underline{r(\theta)}$

$$x = r\cos\theta \rightarrow dx = (r'\cos\theta - r\sin\theta)\,d\theta$$
$$y = r\sin\theta \rightarrow dy = (r'\sin\theta + r\cos\theta)\,d\theta$$



$$L = \sqrt{(dx)^2 + (dy)^2}$$

$$L = \left((r'\cos\theta - r\sin\theta)^2 (d\theta)^2 + (r'\sin\theta + r\cos\theta)^2 (d\theta)^2 \right)^{1/2}$$

$\sqrt{d\theta^2} = |d\theta| = d\theta$

$$L = \left(\underline{\underline{(r')^2\cos^2\theta}} \cancel{- 2rr'\cos\theta\sin\theta} + r^2\sin^2\theta + \underline{\underline{(r')^2\sin^2\theta}} \cancel{+ 2rr'\cos\theta\sin\theta} + \underline{\underline{r^2\cos^2\theta}} \right)^{1/2} d\theta$$

$$L = \boxed{\sqrt{(r')^2 + (r)^2}\,d\theta}$$

$$\boxed{\text{arc length} = \int_\alpha^\beta \sqrt{(r')^2 + (r)^2}\,d\theta}$$

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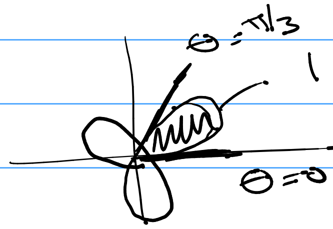
ex's $\quad r = \underline{\sin 3\theta} \qquad \sin 3\theta = 0$



1 petal: area? arc length?

$3\theta = 0, \pi, 2\pi, 3\pi, \ldots$

$\theta = \boxed{0, \pi/3}, \frac{2\pi}{3}, \pi,$

$r = \sin 3\theta$

$r' = 3\cos 3\theta$

area swept out $= \displaystyle\int_0^{\pi/3} \frac{1}{2}[\sin(3\theta)]^2 \, d\theta$

arc length $= \displaystyle\int_0^{\pi/3} \sqrt{(3\cos(3\theta))^2 + (\sin(3\theta))^2} \, d\theta$

$$= \int_0^{\pi/3} \sqrt{9\cos^2 3\theta + \sin^2 3\theta} \, d\theta$$

$8\cos^2 3\theta + \cos^2 3\theta$

$$= \int_0^{\pi/3} \sqrt{1 + 8\boxed{\cos^2 3\theta}} \, d\theta$$

$\frac{1}{2}(1+\cos 6\theta)$

$$= \int_0^{\pi/3} \sqrt{5 + 4\cos 6\theta} \, d\theta$$

$$\int_0 ?$$

$u = 5 + 4\boxed{\cos 6\theta}$?

$du = -24\boxed{\sin 6\theta} \, d\theta$

$\dfrac{du}{\sin 6\theta}$

$\sin 6\theta =$

(ex) area $\quad r(\theta) = (1 + \cos^2(5\theta))^{1/2}$

$$\text{Area} = \int_\alpha^\beta \frac{1}{2}(r)^2 \, d\theta$$

$$\text{Area} = \int_\alpha^\beta \frac{1}{2}(1 + \cos^2(5\theta)) \, d\theta$$

$\boxed{\text{Graph}}$ $r=\sqrt{1+\cos^2(5\theta)}$ $\cdot\ \cos^2(5\theta) = \frac{1+\boxed{\cos(10\theta)}}{2}$