Math 213

~~Q~~ tangent → slope? point? (5, 3)

$x(t) = 5 + \ln(t)$ @ (5, 3) → $t = 1$

$y(t) = t^2 + 2$

$3 = t^2 + 2$

$t^2 = 1$

$t = \pm 1$

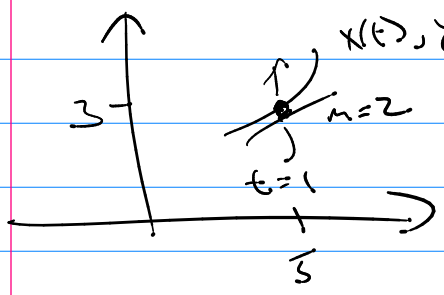
$$\text{slope} = \frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{2t}{1/t} = 2t^2$$

slope @ $t = 1$ slope = 2

pt-slope for a line: $y - y_0 = m(x - x_0)$

$y - 3 = 2(x - 5)$

$\boxed{y = 2x - 7}$



Concavity? $\frac{d^2y}{dx^2}$

$\frac{dy}{dx} = 2t^2$; $y = t^2 + 2$; $dy/dt = 2t$

$x = 5 + \ln(t)$; $dx/dt = \frac{1}{t}$

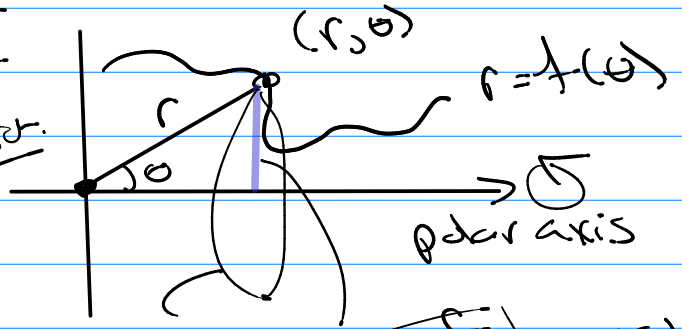
$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left[\frac{dy}{dx}\right] = \frac{\frac{d}{dt}[2t^2]}{dx/dt} = \frac{4t}{1/t} = 4t^2$$

10.4 Applications of polar eqn's ($r = f(\theta)$)

Area Swept Out $= \int_{\alpha}^{\beta} \frac{1}{2}[r]^2 \, d\theta$ Note: $r = f(\theta)$

Arc Length $= \int_{\alpha}^{\beta} \sqrt{(r)^2 + (r')^2} \, d\theta$ Note: $r = f(\theta)$, $r' = f'(\theta)$

Surface area of rotation:



$SA = \int_{\text{start}}^{\text{end}} (\text{circ. circle of rotation})(\text{arc length})$

circ. $= 2\pi(r\sin\theta)$ (radius of rotation $= r\sin\theta$)

So $SA = \int_{\alpha}^{\beta} 2\pi(r\sin\theta)\sqrt{(r)^2 + (r')^2} \, d\theta$

Note: $r = f(\theta)$

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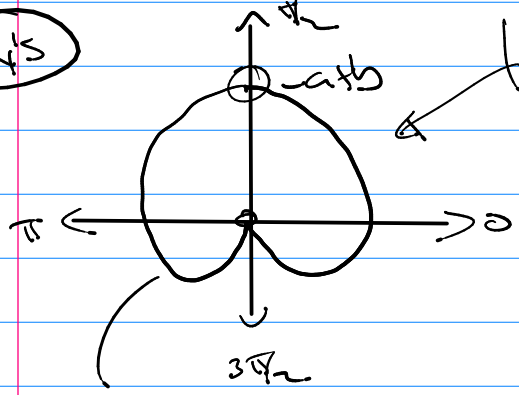
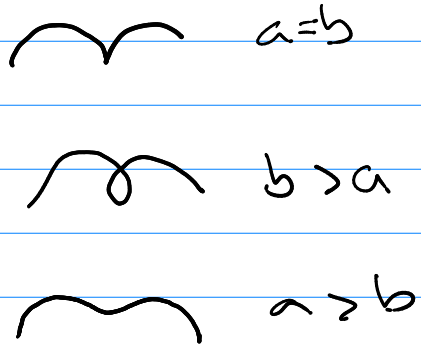
ex's



$r = a + b\sin\theta$

$r = a + b\cos\theta$

$r = a + a\sin\theta$



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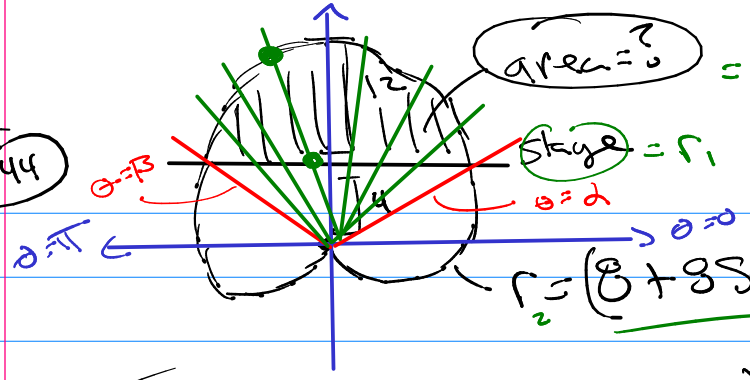


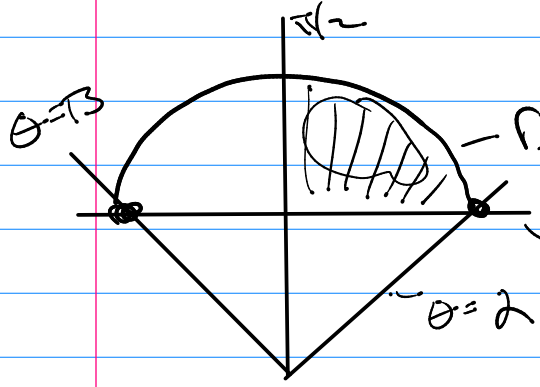
area = ? $= \int_{\alpha}^{\beta} \frac{1}{2}[r_2]^2 - \frac{1}{2}[r_1]^2 \, d\theta$

Stage 2 (eqn's) → cartesian: $y = 4$

→ polar: $r\sin\theta = 4$

$r = \left(\frac{4}{\sin\theta}\right)$

$$A = \int_{\alpha}^{\beta} \frac{1}{2}[8+8\sin\theta]^2 - \frac{1}{2}\left[\frac{4}{\sin\theta}\right]^2 d\theta$$



$r_1 = r_2$

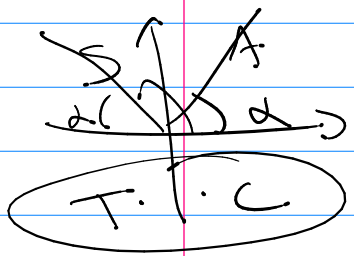
$\frac{4}{\sin\theta} = 8 + 8\sin\theta$

so $\frac{1}{\sin\theta} = 2 + 2\sin\theta \Rightarrow 1 = 2\sin\theta + 2\sin^2\theta$

$\Rightarrow 2\sin^2\theta + 2\sin\theta - 1 = 0$

$$\sin\theta = \frac{-2 \pm \sqrt{4+8}}{4} = \frac{-2 \pm \sqrt{12}}{4}$$

$$\sin\theta = -\frac{1}{2} \pm \frac{1}{2}\sqrt{3}$$



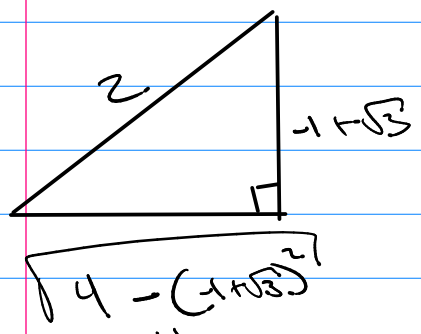
$\sin\theta = \frac{-1+\sqrt{3}}{2}$ $\alpha = \theta = \sin^{-1}\left(\frac{-1+\sqrt{3}}{2}\right)$

~~$\sin\theta = \frac{-1-\sqrt{3}}{2}$~~ $\theta = \sin^{-1}\left(\frac{-1-\sqrt{3}}{2}\right)$

$\theta = \pi - \alpha \qquad \alpha = \sin^{-1}\left(\frac{-1+\sqrt{3}}{2}\right)$

Note: Do some of the above and get to ...

$$\cot\left(\sin^{-1}\left(\frac{-1+\sqrt{3}}{2}\right)\right) = ?$$



$\sin^{-1}\left(\frac{-1+\sqrt{3}}{2}\right) = \theta$

$\frac{-1+\sqrt{3}}{2} = \sin(\theta)$

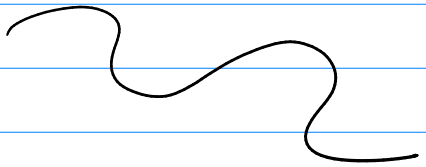
$\sqrt{4-(-1+\sqrt{3})^2} = \sqrt{4-(4-2\sqrt{3})}$

$\sqrt{2\sqrt{3}}$

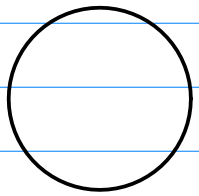
$$\cot\left(\sin^{-1}\left(\frac{-1+\sqrt{3}}{2}\right)\right) = \frac{\sqrt{2\sqrt{3}}}{-1+\sqrt{3}}$$

Analytic Geometry

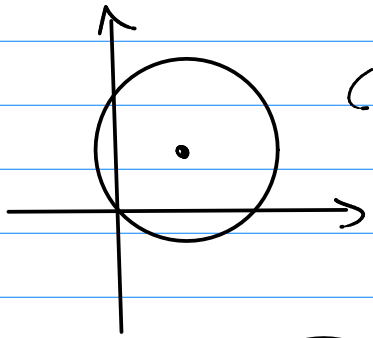
geometry (shapes) → apply coords → get eqn's (functions)

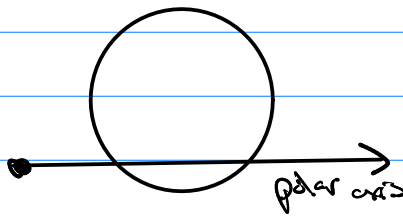


Rx



→ Coord.?



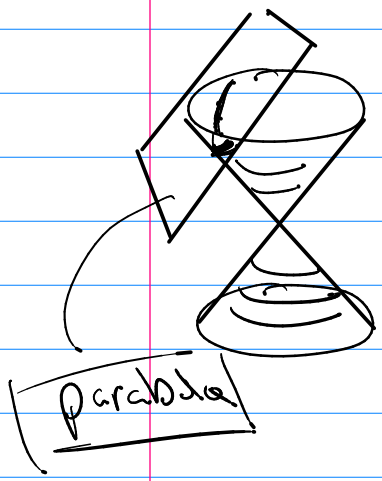


10.5 cartesian

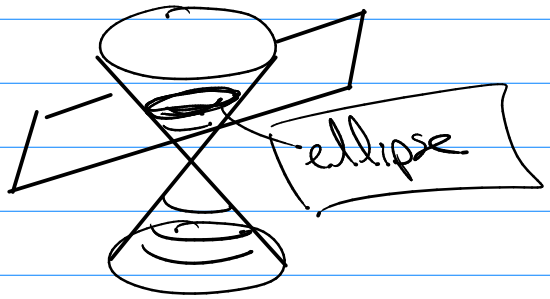
polar 10.6

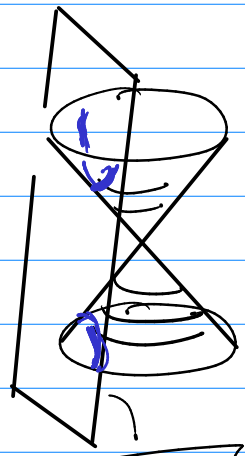
parabola
ellipse (circle)
hyperbola

shapes we will study



upper / lower cone





parabola

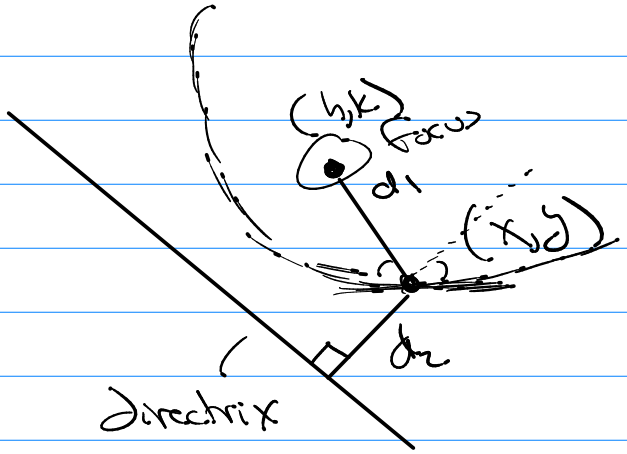
ellipse

hyperbola

conic sections

geometry

Parabola:



$d_1 = d_2$