

# Math 243

Q's Exan 2

10pts possible

→ 10pts extra credit.

$$9 \text{ pts} = 100\%$$

↑  
Kc  $\Rightarrow \sqrt{e^{2x} + 1}$

b)  $\int_{-\infty}^0 \sqrt{1+e^x} dx$

got to limit of type  $\frac{0}{0}$

let  $u = e^x$

$$du = e^x dx$$

or

$$\frac{1}{u} du = dx$$

$$\int \sin^2 x \cos^3 x dx = \int u^2 (1-u^2) du$$

$$u = \sin x dx$$

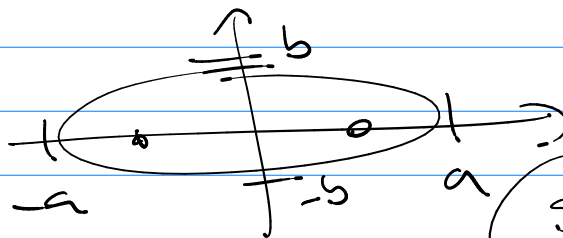
$$du = \cos x dx$$

$$\cos^2 x = 1 - \sin^2 x = 1 - u^2$$

10.5/10.6 Conic Sections (algebraic geometry)

ex eqn of ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

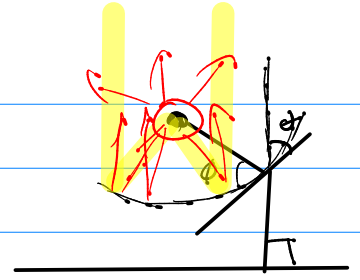
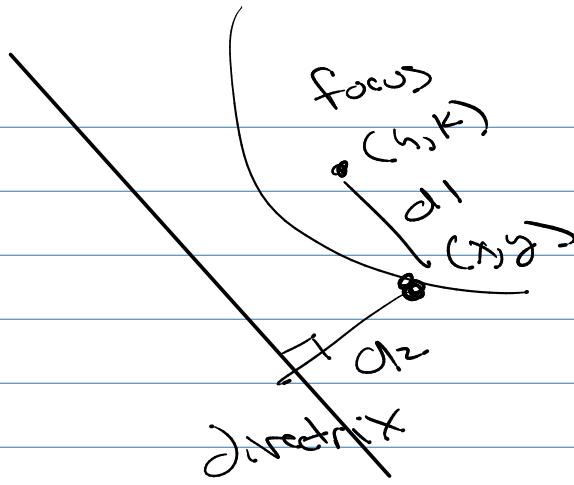


shape

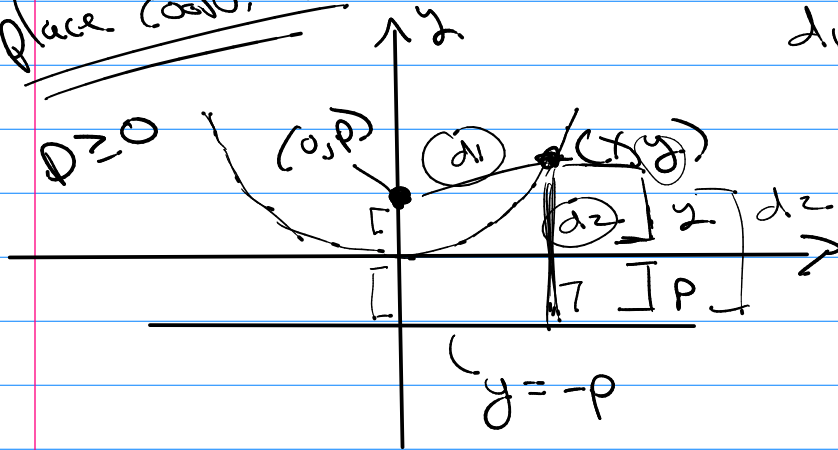
we will go this way.

Parabola

$$d_1 = d_2$$



Place (0,0) axis



$$x^2 + (y-p)^2$$

$$= d_1^2$$

$$= (y+p)^2$$

so  $\sqrt{x^2 + (y-p)^2} = y+p$  eqn of parabola

$$\Rightarrow x^2 + (y-p)^2 = y^2 + 2yp + p^2$$

$$x^2 + y^2 - 2yp + p^2 = y^2 + 2yp + p^2$$

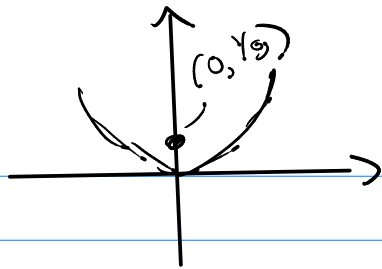
$$x^2 = 4py$$

$$y = \frac{1}{4p} x^2$$

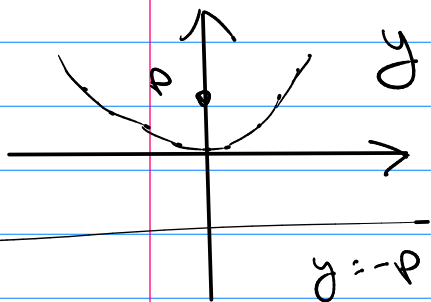
- eqn of parabola
- ① going through origin
- ② opens up
- ③ focus (0, p)

Ex  $y = 2x^2$

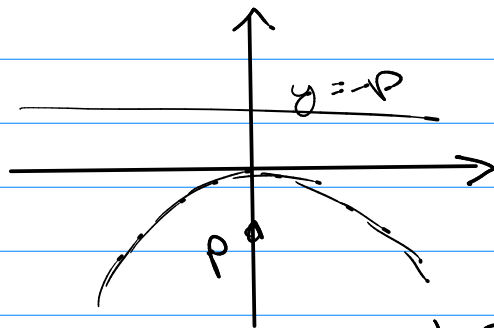
↑  
p



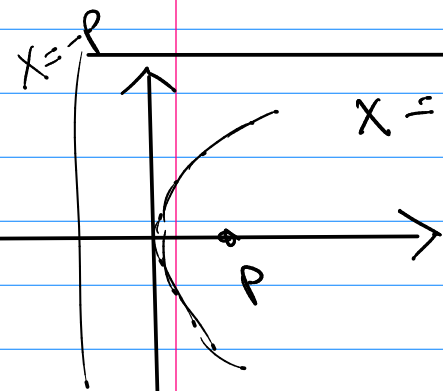
$$\frac{1}{4p} = 2 \rightarrow p = \frac{1}{8}$$



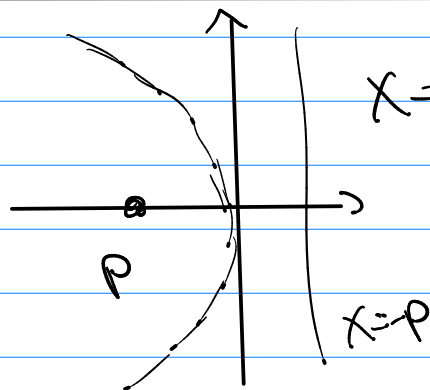
$$y = \frac{1}{4p}x^2, p > 0$$



$$y = \frac{1}{4p}x^2, p < 0$$



$$x = \frac{1}{4p}y^2, p > 0$$



$$x = \frac{1}{4p}y^2, p < 0$$

$$y = 3x^2 + x$$

Parabola

$$\frac{1}{3}y = x^2 + \frac{1}{3}x + \left(\frac{1}{6}\right)^2 - \left(\frac{1}{6}\right)^2$$

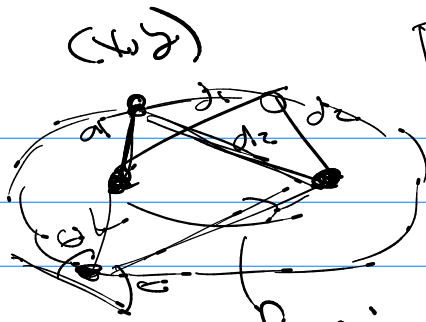
$$\frac{1}{3}y = \left(x + \frac{1}{6}\right)^2 - \frac{1}{36}$$

$$y = 3\left(x + \frac{1}{6}\right)^2 - \frac{1}{12} \rightarrow$$

$p = \frac{1}{12}$

$$y = 3x^2$$

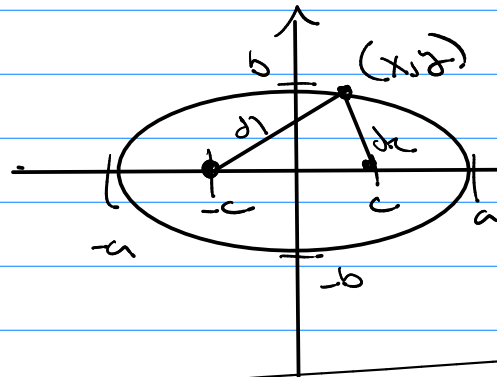
ellipse



$d_1 + d_2 = \text{const.}$

foci.

apply coord



$d_1 + d_2 = 2a$

$d_1 = \sqrt{(x+c)^2 + y^2}$

$d_2 = \sqrt{(x-c)^2 + y^2}$

$c^2 + b^2 = a^2$

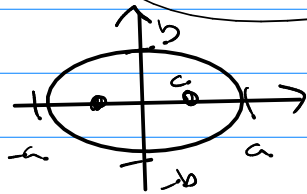
$c^2 = a^2 - b^2$

ellipse  $\rightarrow \sqrt{(x+c)^2 + y^2} + \sqrt{(x-c)^2 + y^2} = 2a$

do algebra

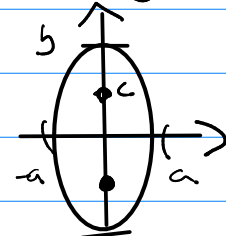
$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

$a \geq b$



$c^2 = a^2 - b^2$

$b \geq a$

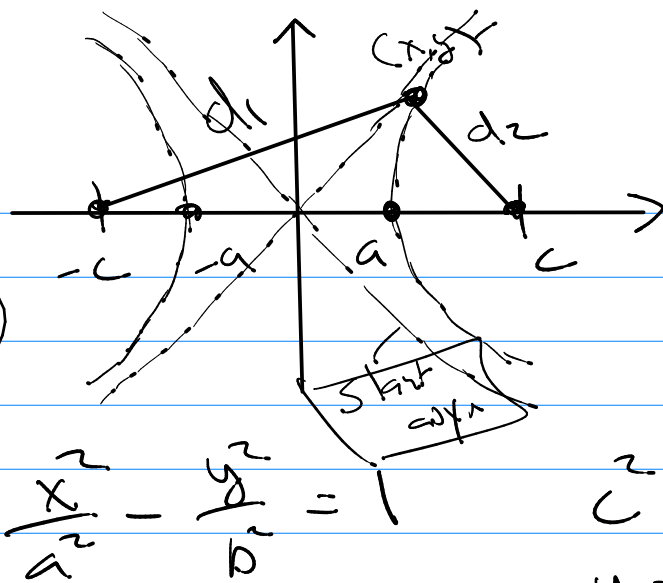


$c^2 = b^2 - a^2$

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

Hyperbolic

Algebra



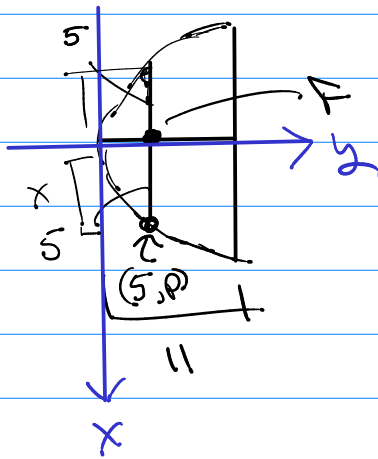
$$d_1 - d_2 = \pm 2a$$

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

$$c^2 = a^2 + b^2$$

$$y = \pm \frac{b}{a} x$$

#50 parabolic reflector



Eqn?

$$y = \frac{1}{4p} x^2 \quad p > 0$$

$$y = p \rightarrow x = 5$$

$$p = \frac{1}{4p} (5)^2 \rightarrow p^2 = \frac{25}{4} \rightarrow p = \sqrt{\frac{5}{2}}$$

$$y = \frac{1}{10} x^2$$