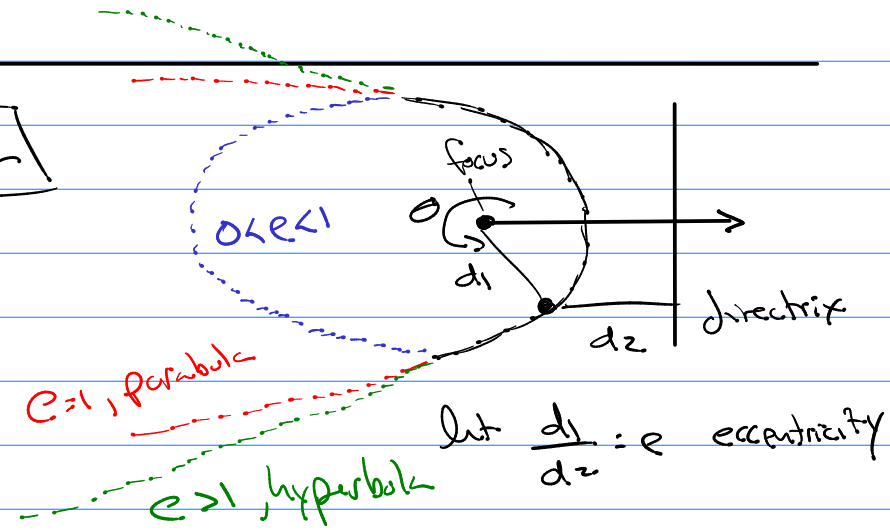
Math 243

Conic Sections: polar eqn

$$r = \frac{ed}{1 \pm e\cos\theta}$$

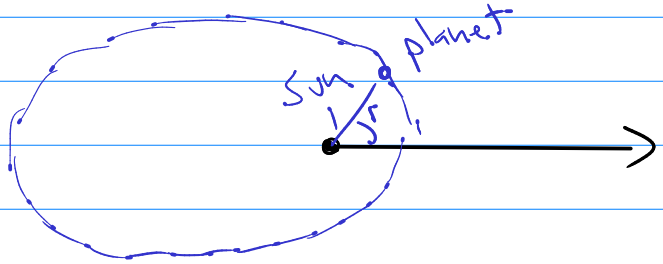
$$r = \frac{ed}{1 \pm e\sin\theta}$$



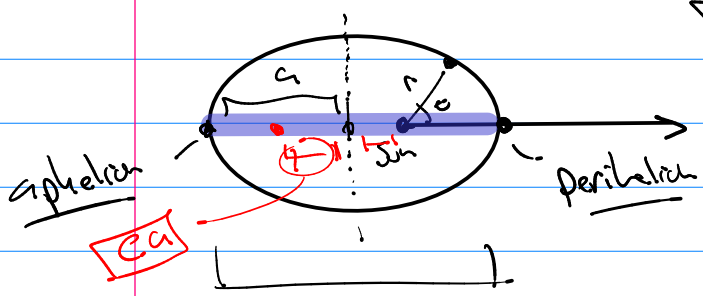
circle: $r = \text{const.}$

Application

Kepler's Laws



① planets revolve around sun in an ellipse with sun at one focus



Major axis = $2a$

Semi-major axis = a

eqn $r = \frac{ed}{1+e\cos\theta}$

↑ cartesian.

$$\frac{\left(x + \frac{e^2 d}{1-e^2}\right)^2}{\left(\frac{ed}{1-e^2}\right)^2} + \frac{y^2}{\left(\frac{ed}{\sqrt{1-e^2}}\right)^2} = 1$$

So $a = \frac{ed}{1-e^2}$

standard eqn $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

$\rightarrow \quad ed = a(1-e^2)$ $\qquad (1-e^2) = (1+e)(1-e)$

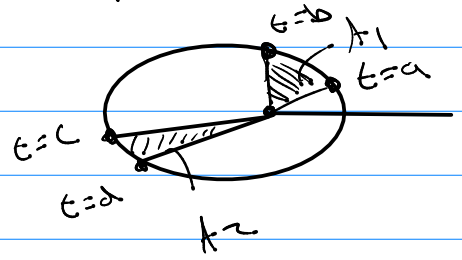
$$r = \frac{ed}{1+e\cos\theta}$$

$$r = \frac{a(1-e^2)}{1+e\cos\theta} \leftarrow$$

distance from sun @ perihelion $r = a(1-e)$

distance from sun @ aphelion $r = a(1+e)$

(2) Sweep equal areas in equal time



$A_1 = A_2$

$\rightarrow (b-a) = (c-d)$ equal time

speed in A_1 > speed in A_2

(3) $(\text{period of revolution})^2 \propto (a)^3$

$P^2 = K a^3$

K: constant

Mercury $\quad e = 0.206$, min dist. from sun is 46×10^7 km

$r = ?$ $\qquad r$ @ $\theta = 0$

$$r = \frac{a(1-e^2)}{1+e\cos\theta} \leftarrow a?$$

r @ $\theta = 0$ $\quad r_{min} = a(1-e) \rightarrow a = \dfrac{46 \times 10^7}{1-.206}$ km

$$a = \frac{9.6 \times 10^7}{.796} \approx 5.8 \times 10^7$$

So $$r = \frac{(5.8 \times 10^7)(1 - (.204)^2)}{1 + .204 \cos\theta}$$

Now: ① max dist. from sun? $r_{max} = a(1+e)$

$$r_{max} = 5.8 \times 10^7 (1.204)$$

② how far does Mercury travel in 1 orbit?

$$\text{arc length} = \int_\alpha^\beta \sqrt{(r')^2 + (r)^2}\, d\theta$$

$$\int_0^{2\pi} \sqrt{(r')^2 + (r)^2}\, d\theta$$ need: $(r = ?_6)^2$, $(r = ?_0)^2$

plug in

Note: can not be integrated by rules.

approximate using .. Simpson's rule.