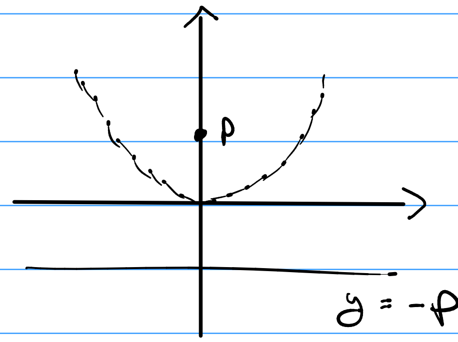
Math 243

Q's Conics in euclidean

Parabola: $y = \boxed{\frac{1}{4p}} x^2$

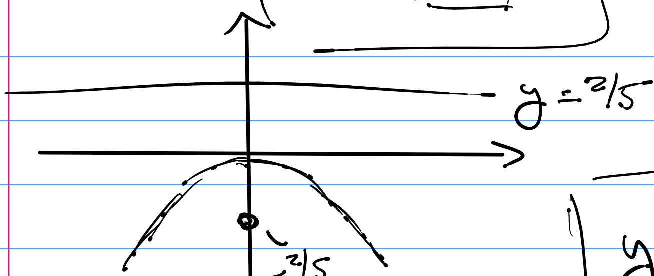


ex $5x^2 + 8y = 0$

$$\boxed{y = \boxed{-\frac{5}{8}} x^2}$$

so $\frac{1}{4p} = -\frac{5}{8}$

$4p = -8/5 \rightarrow p = -2/5$



$$\boxed{y = \frac{1}{4p}(x - h)^2 + k}$$

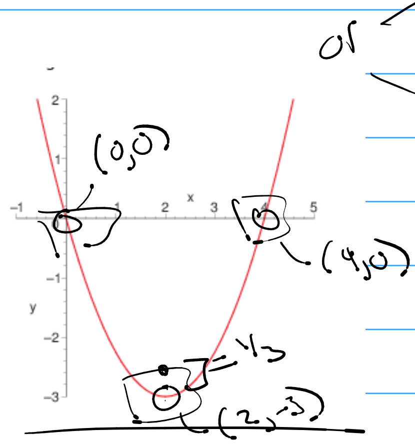
or

$$\boxed{y = ax^2 + bx + c}$$

or

$$\boxed{y = a(x-0)(x-4)}$$

ex



$(0,0) \rightarrow x=0, y=0 \rightarrow c=0$

$y = ax^2 + bx$

$(4,0) \rightarrow$ $16a + 4b = 0$

$(2,-3) \rightarrow$ $4a + 2b = -3$

$8a + 2b = 0 \rightarrow \quad b = -4a$

$-(4a + 2b = -3)$

$4a = 3 \rightarrow a = 3/4 \rightarrow b = -3$

$$\rightarrow \boxed{y = 3/4 x^2 - 3x}$$

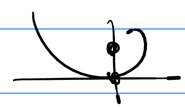
$$\frac{4}{3}y = (x^2 - 4x + 4) - 4$$

$$\frac{4}{3}y = (x-2)^2 - 4$$

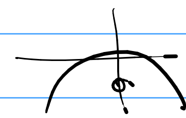
$$y = \boxed{\frac{3}{4}}(x-2)^2 - 3$$

$$" \frac{1}{4p} \rightarrow 4p = \frac{4}{3} \quad p = 1/3$$

$$y = \frac{1}{4p}x^2$$

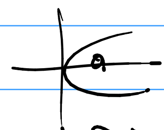


$p > 0$ or

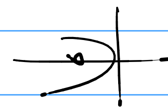


$p < 0$

$$x = \frac{1}{4p}y^2$$

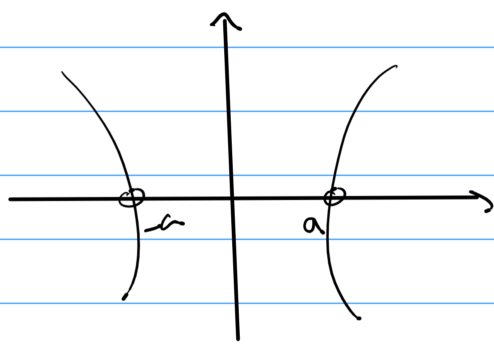


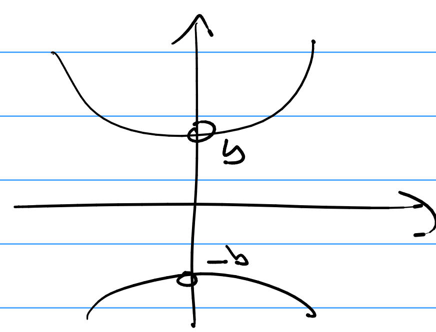
$p > 0$ or



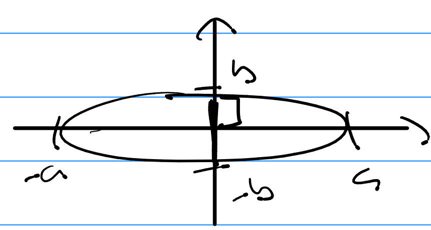
$p < 0$

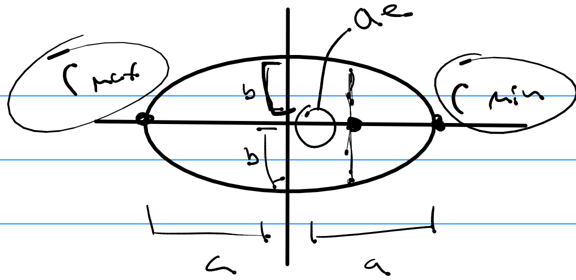
$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \qquad \frac{y^2}{b^2} - \frac{x^2}{a^2} = 1$$





$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$





$$r = \frac{ed}{1+e\cos\theta} \qquad a(1-e^2)$$

$$\frac{r_{max}+r_{min}}{2} = \frac{a(1+e)+a(1-e)}{2} = a$$

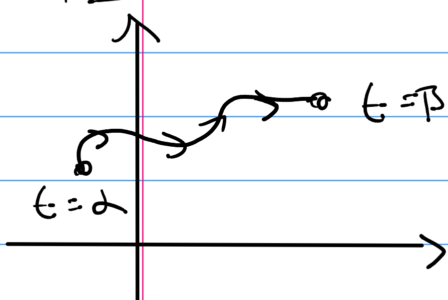
$$(r_{max}\cdot r_{min})^{1/2} =$$

$$\left(\frac{\frac{1}{r_{max}}+\frac{1}{r_{min}}}{2}\right)^{-1} = ed$$

Exam 3 (Friday)

11 probs @ 10pts → 100pts = 100%

ch10 10.1–10.2 Parametric eqns (4 probs)



$x(t), y(t)$

① Sketch ($(x,y) \to x(t), y(t)$ or $x(t), y(t) \to y = f(x)$)

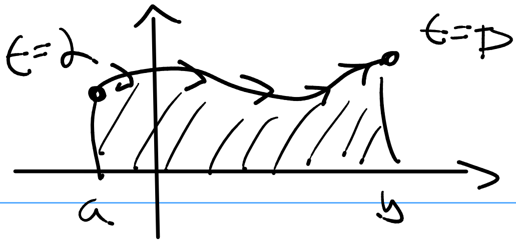
② Derivatives prob.

given $x(t), y(t)$ → slope?
→ curvature?

slope → $\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$

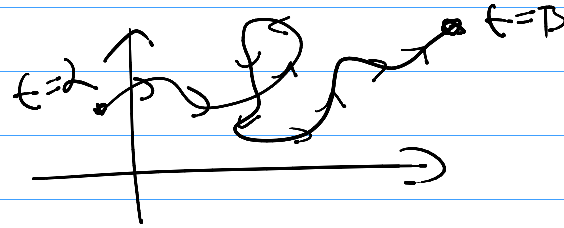
curvature → $\frac{d^2y}{dx^2} = \frac{\frac{d}{dt}\left[\frac{dy}{dx}\right]}{dx/dt}$

③ Area problem



$$A = \int_{\alpha}^{\beta} y(t)\, x'(t)\, dt$$

④ Arc length



$$AL = \int_{\alpha}^{\beta} \sqrt{(x')^2 + (y')^2}\, dt$$

10.3 – 10.4 Polar

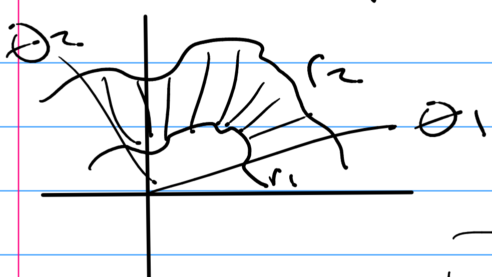
(4 probs)

$x = r\cos\theta$, $y = r\sin\theta$ where $r = f(\theta)$

① sketch

② derivatives $\dfrac{dy}{dx} = \dfrac{dy/d\theta}{dx/d\theta}$

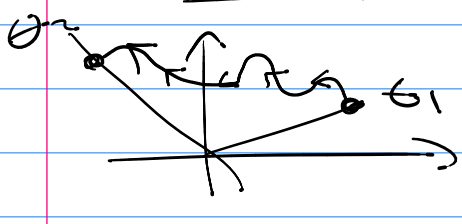
③ Area swept out



$$A = \int_{\theta_1}^{\theta_2} \tfrac{1}{2} r_2^2 - \tfrac{1}{2} r_1^2 \, d\theta$$

④ Arc length

$$AL = \int_{\theta_1}^{\theta_2} \sqrt{(r')^2 + (r)^2}\, d\theta$$



10.5 - 10.6 Conics (3 probs)

① "Some" rectangular x, y problem (parabola or ellipse)

② "Some" polar r, θ problem

$$r = \frac{4}{2 + \cos\theta}$$

③ Kepler's problem