Math 243

11.1 **Sequence:** $f(n)$ = expression for n's

f: Counting Numbers $\to$ Reals

$f: \mathbb{Z}^+ \to \mathbb{R}$

ex) $f(n) = n^2 + 1$, $n = 1, 2, 3, 4, \ldots$

$f(1) = 2$, $f(2) = 5$, $f(3) = 10$, ...

Seq $\{2, 5, 10, \ldots\}$

Note: Numbers: (sets)

$1, 2, 3, \ldots = \mathbb{Z}^+$

$0, 1, 2, 3, \ldots$ = non-negatives

$\ldots, -2, -1, 0, 1, 2, \ldots = \mathbb{Z}$ (integers)

rational $= \left\{ \frac{a}{b} \mid a \in \mathbb{Z}, b \in \mathbb{Z}, b \neq 0, \text{ no common factors} \right\}$

as decimal
(1) terminate
or (2) repeat

such that

Reals: Any decimal number.

reals = rationals — non-rationals (irrationals)

0.25

$0.\boxed{10}101010\ldots$

$\frac{a}{b} \neq 0.121221222122221\ldots$

Seq $\{a_1, a_2, a_3, \ldots\} = \{a_n\}$ for $n = 1, 2, 3 \ldots$

function → rule to generate the seq.

① Closed rule.

$\{2n+7\}$ $n = 1, 2, 3, \ldots$

$\{9, 11, 13, 15, \ldots\}$

useful about closed rules? $a_{100} = 2 \cdot 100 + 7 = \boxed{207}$

② open / recursive / inductive rule

a) basis → state the start value(s)
b) Rule: states how to get new values from old values.

ex $\{7, 9, 11, 13, 15, \ldots\}$

$a_1 = 7$

open rule: $\boxed{a_{n+1} = a_n + 2}$, $n = 1, 2, 3,$

run the rule $a_1 = 7$

rule
$n=1$ $a_2 = a_1 + 2 = 7 + 2 = 9$
$n=2$ $a_3 = a_2 + 2 = 9 + 2 = 11$
$\vdots$

ex **Fibonacci Seq.** f_n, $n = 0, 1, 2, 3, \dots$

basis: $\boxed{\begin{array}{l} f_0 = 0 \\ f_1 = 1 \end{array}}$

inductive: $f_{n+1} = f_n + f_{n-1}$, $n = 1, 2, 3, \dots$

Seq: $\{0, 1, 1, 2, 3, 5, 8, 13, \dots\}$

Applications Seq

(1) Seq $\to$ rule — open? closed?

(2) open rule $\to$ closed rule

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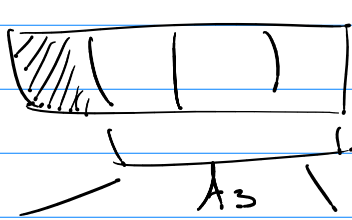
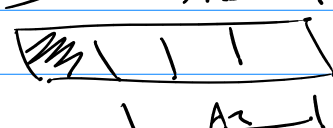
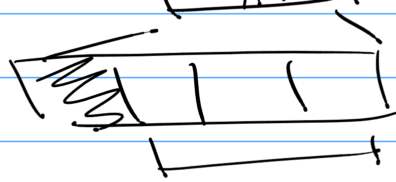
ex **Seq → rule:**

(1) 4 kids get all the cookies in town.

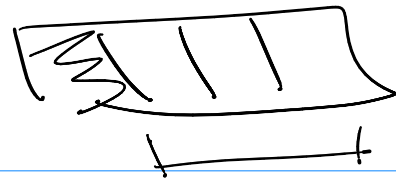
(2) share them tomorrow morning.

(3) a) $A_4 =$  $+1$

b) $A_3 =$  $+1$

c) A_2  $+1$

a) A_1

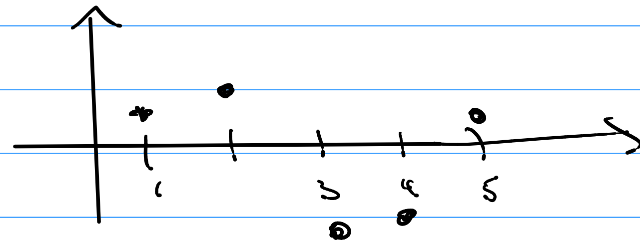


+1

→ Next morning $A_0 = \boxed{240}$

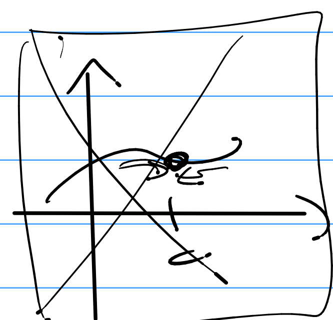
each gets 60 cookies (+) what they had.

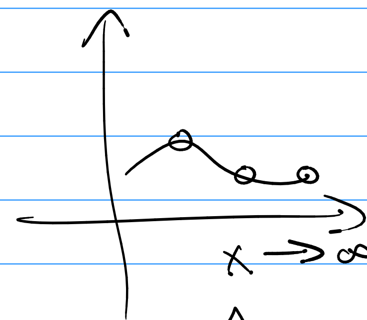
Study: $f: \mathbb{Z}^+ \to \mathbb{R}$



① $\boxed{\text{limit}}$

before $f: \mathbb{R} \to \mathbb{R}$

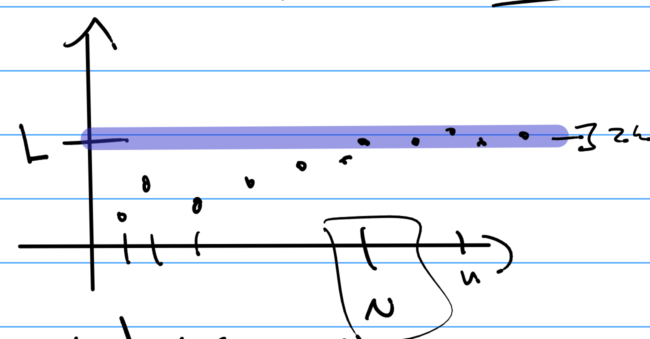




$\lim_{x \to c} f(x)$ doesn't apply

seems ok to apply

Def $\boxed{\lim_{n \to \infty} a_n = L}$



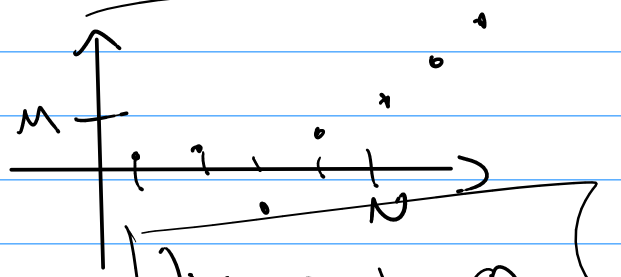
$\forall \varepsilon > 0 \ \exists N > 0$

such that $n > N$ then $|a_n - L| < \varepsilon$

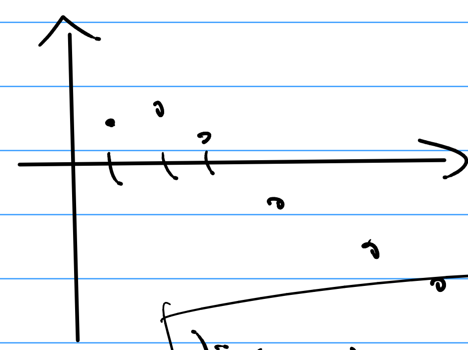
If true we call $\{a_n\}$ convergent

If not we say $\{a_n\}$ is divergent

ex divergent seq's

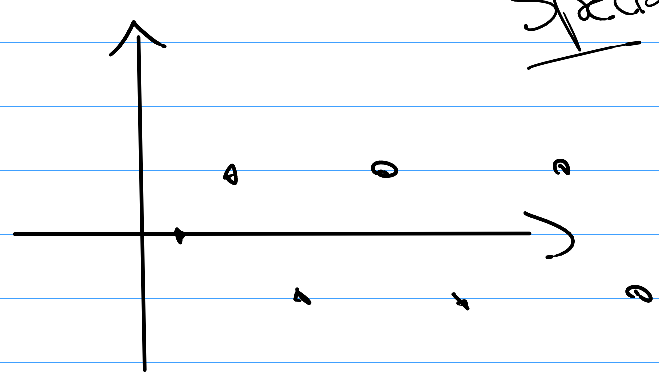


diverges to ∞



diverges to $-\infty$

Special divergent seq's



diverges.

Def $\lim_{n\to+\infty} a_n = +\infty$ (divergent)

$\forall M \ \exists N$ such that $n > N \Rightarrow a_n > M$

$\lim_{n\to+\infty} a_n = -\infty$ (divergent)

$\forall M \ \exists N$ such that $n > N \Rightarrow a_n < M$

Properties of Convergent seq's

given $\{a_n\}$, $\{b_n\}$ are convergent

① $\lim_{n\to\infty}(a_n \pm b_n) = \lim_{n\to\infty} a_n \pm \lim_{n\to\infty} b_n$

$$a_n = \{a_1, a_2, \ldots\}$$
$$b_n = \{b_1, b_2, \ldots\}$$

consider $a_n \pm b_n = \{a_1 \pm b_1, a_2 \pm b_2, \ldots\}$

doing it wrong..

$\boxed{a_n}$ $\{-1, 0, -1, 0, -1, 0, \ldots\}$ div.

$\boxed{b_n}$ $\{1, 0, 1, 0, 1, 0, \ldots\}$ div

$a_n + b_n = \{0, 0, 0, 0, \ldots\}$ $\boxed{\text{conv.}}$

Cont. 11.1 Next class