Math 243

Q's Webassign? → Yes, ... I need to get next homeworks up.

---

basis: $A_0 = 240$

rule: $\boxed{A_{n+1} = \frac{4}{3} A_n + 1}$

$A_{n+1} = \boxed{\quad}\ + 1$

A_n

$240, \boxed{321}, \boxed{429}, \boxed{573}, \boxed{765}$

→ $4 \cdot 80 + 1$

per kid? 60 each

$K_1 = 80 + 60$
$K_2 = 107 + 60$
$K_3 = 143 + 60$
$K_4 = 191 + 60$

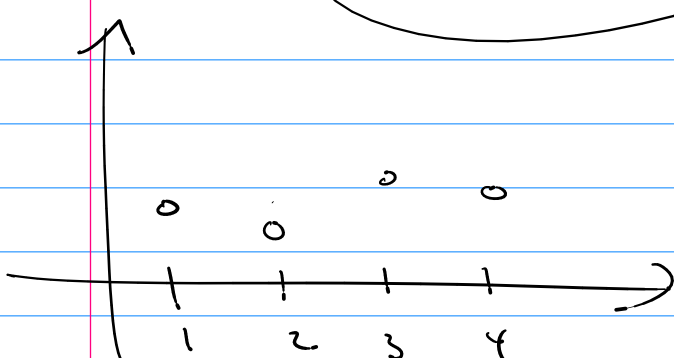
$\boxed{11.1}$ **Seq.** $\{a_1, a_2, \dots\} = \{a_n\}$ $n = 1, 2, 3, \dots$

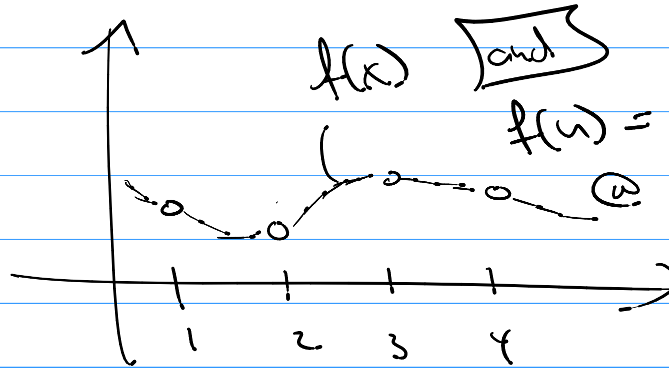
$\boxed{\text{Convergence}}$

$$\lim_{n \to \infty} a_n = L$$

← how to find L?

consider $\{a_n\}$, $f(x)$



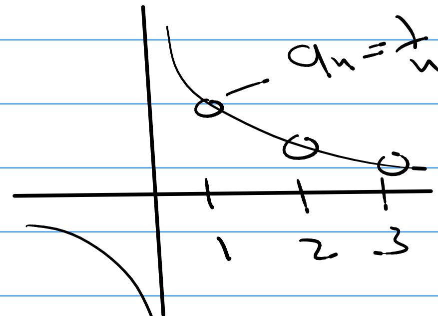


$f(x)$ $\boxed{\text{and}}$ $f(n) = a_n$ @ integers

Thm Given $\{a_n\}$, $f(x)$ and $f(n) = a_n$ @ integers.

If $\lim\limits_{x\to+\infty} f(x) = L \rightarrow \lim\limits_{n\to+\infty} a_n = L$

ex $\lim\limits_{n\to+\infty} \frac{1}{n}$ <u>but</u> $f(x) = \frac{1}{x}$



<u>and</u> $f(n) = \frac{1}{n}$

so $\lim\limits_{x\to+\infty} \frac{1}{x} = 0$ so $\boxed{\lim\limits_{n\to+\infty} \frac{1}{n} = 0}$

ex $\lim\limits_{n\to+\infty} \frac{n^2}{e^n}$ Consider $f(x) = \frac{x^2}{e^x}$ while $f(n) = \frac{n^2}{e^n}$

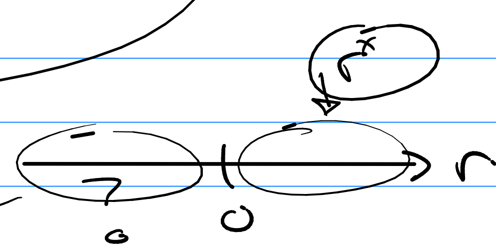
So $\lim\limits_{x\to+\infty} \frac{x^2}{e^x} = \lim\limits_{x\to+\infty} \frac{2x}{e^x} = \lim\limits_{x\to+\infty} \frac{2}{e^x} = 0$

$\therefore \boxed{\lim\limits_{n\to+\infty} \frac{n^2}{e^n} = 0}$

ex $\lim\limits_{n\to+\infty} \boxed{r^n}$ consider $f(x) = r^x$ <u>only</u> defined if $r \geq 0$

$\underline{n = 1, 2, 3, 4, \ldots}$

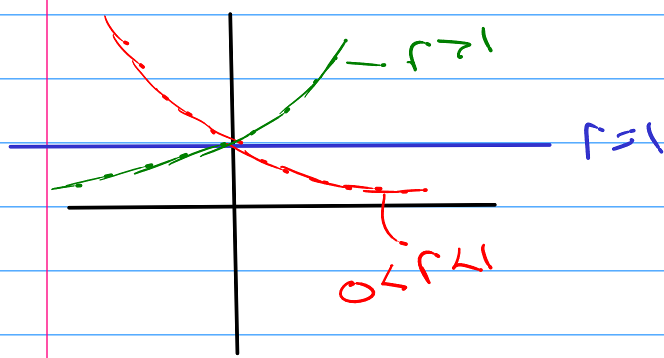
So



$\{r^n\}$ $n = 1, 2, 3, \ldots$

allows $\boxed{\text{all}}$ r's

so $$\lim_{x\to+\infty} r^x = \begin{cases} 1 & \text{if } r=1 \\ 0 & \text{if } r<1 \\ \text{divergent } \infty & \text{if } r>1 \end{cases}$$



if $r=0$, $r<0$

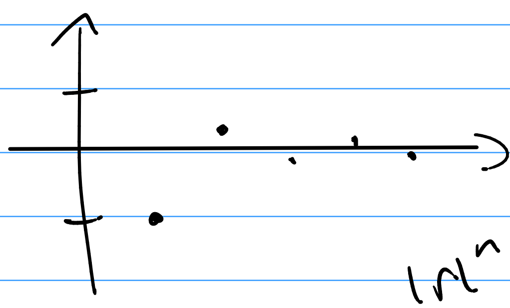
r^x makes no sense!

① consider $r=0$ for $\{0^n\}$ $n=1,2,3,\ldots \rightarrow \{0,0,0,\ldots\}$

$$\lim_{n\to\infty} 0^n = 0$$

② consider $r<0$ so $r^n = \underline{(-1)^n}\,\overset{\text{pos}}{|r|^n}$, $n=1,2,3,\ldots$

$\rightarrow \ -\boxed{|r|^1},\ +\boxed{|r|^2},\ -\boxed{|r|^3},\ +|r|^4,\ \ldots$



$$\lim_{n\to\infty} r^n = 0 \quad \text{if } -1<r<0$$

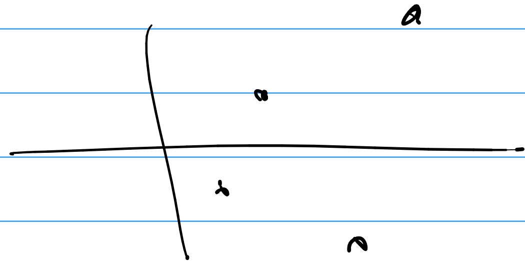
$|r|^n \to 0$

$|r|<1$

$r=-1$ seq $-1, 1, -1, 1, -1, 1, -1, 1, \ldots$ div.

$r<-1$ seq $-|r|, |r|^2, -|r|^3, |r|^4$ div.

$\rightarrow \infty$



$\boxed{\text{So}}$ $\lim_{n\to\infty} r^n = \begin{cases} 0 & -1<r<1 \\ 1 & \text{if } r=1 \\ \underline{\text{div.}} & r\le -1,\ r>1 \end{cases}$

---

if $\{a_n\}$, $\{b_n\}$ are convergent

$\boxed{\text{Properties}}$

① $\lim_{n\to\infty} a_n \pm b_n = \lim_{n\to\infty} a_n \pm \lim_{n\to\infty} b_n$

② $\lim_{n\to\infty} c a_n = c \lim_{n\to\infty} a_n$

③ $\lim_{n\to\infty} a_n b_n = \lim_{n\to\infty} a_n \lim_{n\to\infty} b_n$

④ $\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} a_n / \lim_{n\to\infty} b_n$ $\boxed{\text{except}}$ $\lim_{n\to\infty} b_n \neq 0$

ex $\lim_{n\to\infty} \frac{3n^2+n-2}{2n(n+1)} = \lim_{n\to\infty} \frac{3+\frac{1}{n}-2\left(\frac{1}{n}\right)^2}{2\left(1+\frac{1}{n}\right)} = \boxed{\frac{3}{2}}$

---

⑤ $\lim_{n\to\infty} (a_n)^p = \left(\lim_{n\to\infty} a_n\right)^p$ if $p>0$, $a_n>0$

⑥ $\lim_{n\to\infty} f(a_n) = f\left(\lim_{n\to\infty} a_n\right)$ if f is cont

@ $\lim_{n\to\infty} a_n = L$

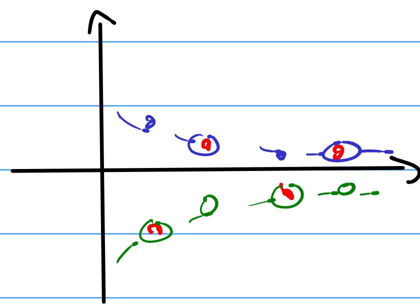
(ex) $\lim_{n\to\infty} \cos\left(\frac{1}{n}\right) = \cos\left(\lim_{n\to\infty} \frac{1}{n}\right) = \cos(0) = 1$

Squeeze thm $\{a_n\}, \{b_n\}, \{c_n\}$

and $a_n \le b_n \le c_n \quad n = 1, 2, 3, \ldots$

Find $\lim_{n\to\infty} a_n = L$, $\lim_{n\to\infty} c_n = L \rightarrow \lim_{n\to\infty} b_n = L$

(ex) $\lim_{n\to\infty} |a_n| = 0$



$\{a_n\}$

$\{|a_n|\}$

$\Rightarrow \lim_{n\to\infty} a_n = 0$

$\Rightarrow$ really says

$\lim_{n\to\infty} -|a_n| = 0$

$\lim_{n\to\infty} |a_n| = 0$

Monotonic / Bounded

Def $\{a_n\}$ is monotonic

1. Increasing $a_{n+1} > a_n$
2. Decreasing $a_{n+1} < a_n$

(ex) harmonic seq $\left\{\frac{1}{n}\right\}$ $n = 1, 2, 3, \ldots$

$a_n = \frac{1}{n}$ vs $a_{n+1} = \frac{1}{n+1}$

$\frac{1}{n} > \frac{1}{n+1}$ $a_n > a_{n+1}$

so $\frac{1}{n}$ is monotonic dec

ex $\{ \frac{1}{1}, \frac{1}{3}, \frac{1}{2}, \frac{1}{4}, \frac{1}{3}, \frac{1}{5}, \frac{1}{4}, \frac{1}{6}, \frac{1}{5}, \frac{1}{7}, \frac{1}{6}, \frac{1}{8}, \ldots \}$

$+ \quad - \quad + \quad - \quad + \quad -$

not monotonic

Def Bounded

there exists

① bounded above if $\exists M$ such that $a_n \le M$ for all a_n

② bounded below if $\exists M$ such that $a_n \ge M$ for all a_n

inc. and bounded above

dec. and bounded below

Thm if $\{a_n\}$ that is monotonic and bounded it is convergent.
