Math 243

Seq $\{a_n\}$ $n=1,2,3,\dots$ $\{a_1, a_2, a_3, \dots\}$

We can: ① $\lim_{n\to\infty} a_n = L$

② convergence

③ ex convergent seq

11.2 Add the terms of a seq. (Sum)

ex puzzle: day 1 $= 1 = a_1$

day 2 $= 2\cdot a_1 = 2\cdot 1 = 2 = a_2$

day 3 $= a_3 = 2\cdot a_2 = 2\cdot 2 = 4$

Seq: $a_1 = 1$, $a_{n+1} = 2\cdot a_n$ $n = 1, 2, \dots, 31$

total: $\boxed{a_1 + a_2 + a_3 + \dots + a_{31}}$ = Sum

Notation: $\sum_{n=1}^{31} a_n = a_1 + a_2 + \dots + a_{31}$

upper bound

seq

lower bound

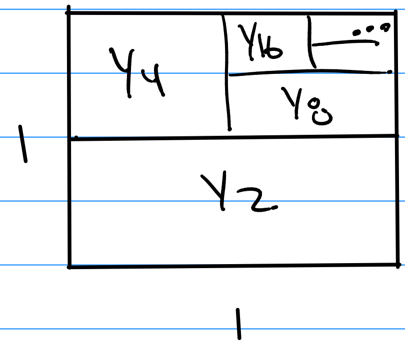
Solve: seq: $1, 2, 4, 8, 16, 32, \dots, 2^{(n-1)}, \dots, 2^{30}$

n=1 n=2 n=3 n=4 n=5

Sum: $1 + 2 + 4 + 8 + 16 + 32 + \dots + 2^{30} = 2^{31} - 1$

Infinite Series

$$\sum_{n=1}^{\infty} a_n = a_1 + a_2 + a_3 + \dots$$



Area = 1

$$\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \dots = 1$$

$$1 + 1 + 1 + 1 + \dots$$ not finite, divergent

How to work with $\sum_{n=1}^{\infty} a_n = a_1 + a_2 + a_3 + \dots$?

Partial Sums:

Seq

$$
\begin{aligned}
S_1 &= a_1 \\
S_2 &= a_1 + a_2 \\
S_3 &= a_1 + a_2 + a_3 \\
&\vdots \\
S_k &= a_1 + a_2 + a_3 + \dots + a_k
\end{aligned}
$$

$$\vdots$$

$$S = \sum_{n=1}^{\infty} a_n = a_1 + a_2 + a_3 + \dots$$

goal $\boxed{\lim_{k \to \infty} S_k = S}$ limit of the partial sums to be the infinite series.

Def $\{S_n\}$, seq. of partial sums

If the seq. is convergent and $\lim_{n \to \infty} S_n = S$ a real number

then: $\sum_{n=1}^{\infty} a_n = S$ (call $\sum_{n=1}^{\infty} a_n$ convergent) ex $\quad 0.99999\ldots = \frac{9}{10} + \frac{9}{100} + \frac{9}{1000} + \ldots = S$

$$S = \frac{9}{10} + \frac{9}{100} + \frac{9}{1000} + \ldots$$

$$10S = 9 + \frac{9}{10} + \frac{9}{100} + \frac{9}{1000} + \ldots$$

$$10S - S = 9S = 9$$

$\rightarrow \boxed{S = 1} \qquad 0.999\ldots = 1.000\ldots$

$\frac{1}{4} = 0.25 = 0.249999\ldots$

Series to know

① Geometric Series $\quad \sum_{n=0}^{\infty} ar^n = a + ar + ar^2 + \ldots$

$r \neq 0$

$$\boxed{\sum_{n=0}^{\infty} ar^n \overset{?}{=} S}$$

$\rightarrow$ what if $r = 1$

$$\sum_{n=0}^{\infty} a(1)^n = a + a + a + a + \ldots = \text{div.}$$

$S_1 = a$

$S_2 = a + a = 2a$

$S_3 = a + a + a = 3a$

$\vdots$

$S_n = n a$

$n \to \infty \quad S_n \to \text{div.}$

$\to$ $r=-1$? $\sum_{n=0}^{\infty} a(-1)^n = a - a + a - a + a - a + \dots$

$S_1 = a$

$S_2 = 0$

$S_3 = a$

$S_4 = 0$

$\vdots$

$S_n = 0$ or a $\boxed{\text{divergent}}$

assume conv.

<u>try:</u> $S = a + ar + ar^2 + ar^3 + \dots$

$rS = ar + ar^2 + ar^3 + \dots$

$S - rS = a$

$S(1-r) = a$

$$\boxed{S = \frac{a}{1-r}}$$ if <u>convergent</u> (<u>is it?</u>)

is it convergent? <u>when?</u>

$a + ar + ar^2 + \dots$

$S_1 = a$

$S_2 = a + ar$

$S_3 = a + ar + ar^2$

$\vdots$

$S_k = a + \underline{ar + ar^2 + \dots + ar^k}$

consider $rS_K = \boxed{ar + ar^2 + \ldots + ar^K} + ar^{K+1}$

So $S_K - rS_K = a - ar^{K+1}$

$$S_K = a\left(\frac{1-r^{K+1}}{1-r}\right)$$

$r^{K+1} \to 0$ if $|r|<1$

$\lim_{K\to\infty} S_K = S$ conv. of seq. So $\lim_{K\to\infty} a\left(\frac{1-r^{K+1}}{1-r}\right) = \frac{a}{1-r}$

$$\therefore \boxed{\sum_{n=0}^{\infty} ar^n = \frac{a}{1-r} \text{ if } |r|<1 \quad \text{diverges otherwise}}$$

and along with this $\sum_{n=0}^{K} ar^n = a\left(\frac{1-r^{K+1}}{1-r}\right)$

$$\sum_{n=0}^{30} (1)2^n = (1)\left(\frac{1-2^{31}}{1-2}\right) = 2^{31} - 1$$

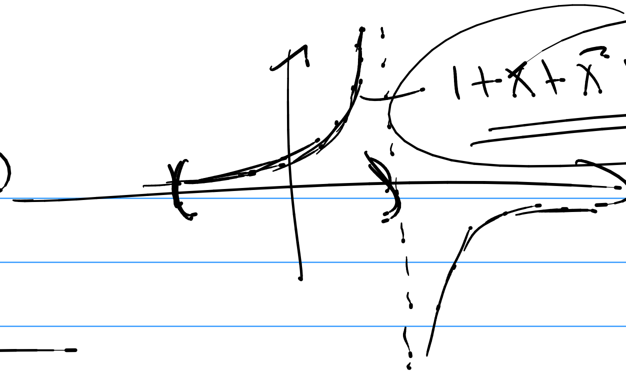
Consider: $1 + x + x^2 + x^3 + \ldots = \frac{1}{1-x}$ when $|x|<1$

$$\sum_{n=0}^{\infty} (1)(x)^n = \frac{1}{1-x}$$

$a \quad r^n$

$|x|<1$

$$\frac{1}{1-x} = -\frac{1}{(x-1)}$$



$$1 + x + x^2 + x^3 + \dots$$