Math 243

$\sum a_n$ div.? conv.?

no test: ① $S_k = \sum_{n=1}^{k} a_k = a_1 + a_2 + \dots + a_k$ (partial sum)

② $\lim_{k\to\infty} S_k = S \Rightarrow \sum a_n = S$

Tests ① div. test: if $\lim_{n\to\infty} a_n \neq 0 \rightarrow \sum a_n$ div.

② Integral test: $f(x)$ such that $f(n) = a_n$

it is ① cont ② dec ③ always pos.

$\int_1^\infty f(x)\,dx$ → div then $\sum a_n$ div.

→ conv then $\sum a_n$ conv.

③ p-series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ → conv $p > 1$

→ div. $p \leq 1$

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to now we have: ① $\sum_{n=0}^{\infty} a r^n = \frac{a}{1-r}$ if $|r| < 1$

(for conv. series) ② $\sum_{n=1}^{\infty} \frac{1}{n^p}$ → conv if $p > 1$

→ div if $p \leq 1$

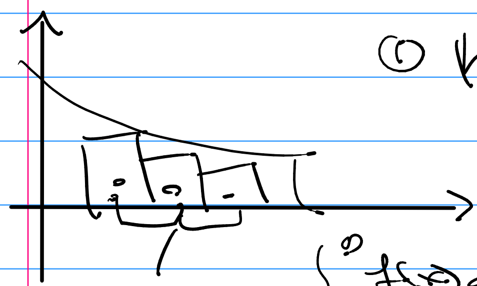
Tests Comparison tests (compare an unknown series to a known series)

① Comparison Test

② limit comparison test

Comparison test is "like" how the integral test was made

(A)



① known thing conv.

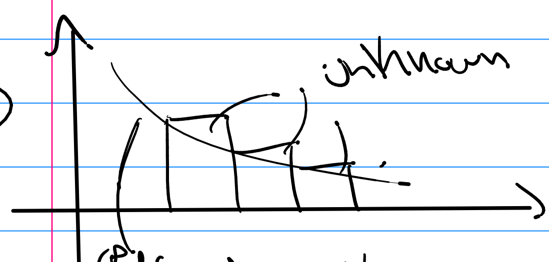
② unknown < known

③ unknown conv.

$\int_1^\infty f(x)\,dx$; known

series is unknown

(D)



① known thing div.

② know < unknown

③ unknown div

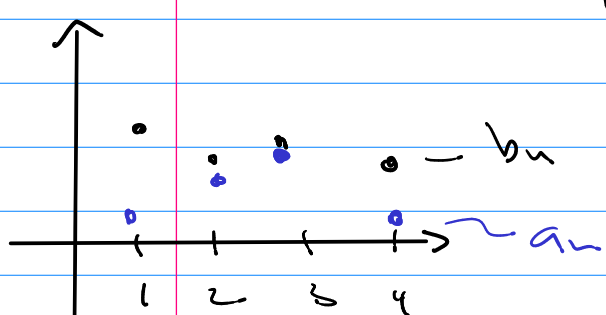
$\int_1^\infty f(x)\,dx$ is known

Comparison test

① $\sum a_n$, $\sum b_n$

② $a_n > 0$, $b_n > 0$

③ $a_n \le b_n$



test

a) if $\sum a_n$ div $\to$ $\sum b_n$ div

b) if $\sum b_n$ conv $\to$ $\sum a_n$ conv

How to use this test? $\sum c_n$ = conv? div?

Step ① Guess

ex $\displaystyle\sum_{n=1}^{\infty} \frac{n+1}{n^3-n+3} \sim \frac{n}{n^3} = \frac{1}{n^2}$ for large n

guess conv.

(ex) $\sum_{n=1}^{\infty} \frac{n^2+1}{n^3-n+3} \sim \frac{n^2}{n^3} = \frac{1}{n}$

for large n

guess div.

Step 2 Use comparison test

guess div? → find a smaller series that div

guess conv? → find a larger series that conv.

inequalities

(ex) $\sum_{n=1}^{\infty} \frac{1}{n^2+1}$, $\frac{1}{n^2+1} \sim \frac{1}{n^2}$ for large n

guess convergent

find larger series that is conv.

$\frac{1}{n^2+1} < \frac{1}{n^2+1-1} = \frac{1}{n^2}$

is conv.

So $\sum_{n=1}^{\infty} \frac{1}{n^2+1}$ is also conv.

(ex)

$\sum_{n=1}^{\infty} \frac{1}{n^7+n^6+n^5+n^4+n^3+n^2+n+1} < \sum_{n=1}^{\infty} \frac{1}{n^7}$ conv.

So $\boxed{\text{conv}}$

(ex) $\sum_{n\to\infty} \frac{6^n}{5^n+1}$ by div test $n\to\infty$ $a_n \to \infty$ $\to \boxed{\text{div}}$

Comparison: (guess div)

$$\frac{6^n}{5^n+1} \boxed{\geq} \frac{6^n}{5^n+5^n} = \frac{6^n}{2(5^n)} = \boxed{\frac{1}{2}\left(\frac{6}{5}\right)^n}$$

ar^n

div. geometric

b/c $\frac{6}{5} > 1$

So $\sum \frac{6^n}{5^n+1}$ is also div.

$\boxed{\text{limit Comparison}}$ $\sum a_n, \sum b_n$ $a_n > 0, b_n > 0$

if $\lim_{n\to\infty} \frac{a_n}{b_n} = C$ a positive finite number

so both conv or both div.

How to use it?

Step 1 Guess!

(ex) $\sum_{n=1}^{\infty} \frac{1}{n^2-1}$ $\frac{1}{n^2-1} \sim \frac{1}{n^2}$ for large n

step 2 use your guess

$$\lim_{n\to\infty} \frac{\frac{1}{n^2-1}}{\frac{1}{n^2}} = \lim_{n\to\infty} \frac{n^2}{n^2-1} = \lim_{n\to\infty} \frac{1}{1-1/n^2} = 1$$

pos. number

So b/c $\sum \frac{1}{n^2}$ conv. $\rightarrow$ $\boxed{\sum \frac{1}{n^2-1} \text{ also conv.}}$