Math 243

Q's

ex $\sum_{n=1}^{\infty} (-1)^n \frac{n}{\sqrt{n^3+7}} = \sum_{n=1}^{\infty} (-1)^n b_n$

alt. series!

$b_n = \frac{n}{\sqrt{n^3+7}}$

Alt. Series test

① $b_n > b_{n+1}$

② $\lim_{n\to\infty} b_n = 0$

if true? conv

① $b_n = \frac{n}{\sqrt{n^3+7}} \overset{?}{>} b_{n+1} = \frac{n+1}{\sqrt{(n+1)^3+7}}$

② $\lim_{n\to\infty} \frac{n}{\sqrt{n^3+7}} = \lim_{n\to\infty} \frac{1/n^{1/2}}{\sqrt{1+7/n^3}} = \frac{0}{\sqrt{1+0}} = 0$

dec? check slope of $f(x) = \frac{x}{\sqrt{x^3+7}}$

$\sum_{n=1}^{\infty} (-1)^n \frac{1}{4^n n!}$ error < 0.00001

alternating!

$S_k = -\frac{1}{4} + \frac{1}{32} - \frac{1}{384} + \dots + (-1)^k \frac{1}{4^k k!}$

$R_k < \frac{1}{4^{k+1}(k+1)!} < 0.0001$

$S_4 = \boxed{-\frac{1}{4} + \frac{1}{32} - \frac{1}{384} + \frac{1}{6144}}$

11.6 Finite Sums $\boxed{a_1 + a_2 + a_3 + a_4}$

we have assoc. and commutative law

$\to$ any rearrangement of $a_1 + a_2 + a_3 + a_4 = S$

$a_4 + a_2 + a_1 + a_3 = S$

Q What about $\sum a_n = S$?

does $a_1 + a_2 + a_3 + a_4 + \ldots = S$ for $\boxed{\text{any}}$ arrangement of a_i?

1) Yes if $\sum a_n$ is absolutely convergent

ex $1 + \frac{1}{4} + \frac{1}{9} + \frac{1}{16} + \ldots = S = \sum_{n=1}^{\infty} \frac{1}{n^2}$

2) no if $\sum a_n$ is cond. convergent

($\sum |a_n|$ div. , $\sum a_n$ conv)

ex $1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \frac{1}{6} + \ldots = S$

Fact for any real number, r, there is some rearrangement of a_i such that $\underset{\text{rearranged}}{\sum a_i} = r$

11.7 Strategy $\to$ pick a test!

1st Know your two basics $\sum \frac{1}{n^p}$, $\sum ar^n$

2nd Know alt. series $\sum (-1)^n b_n$

test

① div test $\sum a_n$ if $\lim_{n\to\infty} a_n \neq 0 \to \sum a_n$ diverges

$$\sum_{n=1}^{\infty} \frac{\sqrt{n^3+1}}{\sqrt[4]{n^4+n^3}} \overset{\text{large } n}{\sim} \frac{n^{3/2}}{n} = n^{1/2}$$

② Integral test $\int_1^\infty f(x)\,dx \to$ conv, then $\sum a_n$ conv.
$\searrow$ div, then $\sum a_n$ div

$\sum a_n \to f(n) = a_n$

$f(x) \to$ properties?

③ Comparison tests

a) Comparison test

b) limit comparison test

ex $\displaystyle\sum_{n=1}^{\infty} \boxed{\frac{\sqrt{n^3+1}}{\sqrt[4]{n^7+1}}}$ this is like (for large n)

$$\sim \frac{\sqrt{n^3}}{\sqrt[4]{n^7}} = \frac{n^{6/4}}{n^{7/4}} = \frac{1}{n^{1/4}}$$

div. p-series

now what? you think $\sum_{n=1}^{\infty} \frac{\sqrt{n^3+1}}{\sqrt[4]{n^7+1}}$ is like $\sum \frac{1}{n^{1/4}}$

(div)

Comparison test $\frac{\sqrt{n^3+1}}{\sqrt[4]{n^7+1}} > \text{div?}$ ← find this.

ex $\frac{\sqrt{n^3+1}}{\sqrt[4]{n^7+1}} > \frac{\sqrt{n^3}}{\sqrt[4]{n^7+1}} > \frac{\sqrt{n^3}}{\sqrt[4]{n^7+n^7}} = \frac{n^{3/2}}{\sqrt[4]{2}\, n^{7/4}}$

$= \boxed{\frac{1}{\sqrt[4]{2}\, n^{1/4}}}$

limit compare $\lim_{n\to\infty} \frac{\left(\frac{\sqrt{n^3+1}}{\sqrt[4]{n^7+1}}\right)}{\left(\frac{1}{n^{1/4}}\right)} = 1$

= finite pos. const

→ they act alike

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ratio test, root test.

$$\lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right| = L$$

or

$$\lim_{n\to\infty} \left(|a_n| \right)^{1/n} = L$$

ex $\sum (-1)^n \frac{1}{2^n}$ is it absolutely conv?

(1) check $\sum \left| (-1)^n \frac{1}{2^n} \right| = \sum \frac{1}{2^n}$ ✓

(2) ratio test
(3) or root test

$= \frac{1}{2}$