Math 243

Q's

Symbols:

$\mathbb{R}$ all reals $\qquad$ $\mathbb{N}$ all naturals

$\mathbb{Q}$ all rationals

$\mathbb{Z}$ all integers

$\in$ element of $\qquad$ $\exists$ there exists

$\ni$ such that $\qquad$ $\forall$ for all

$\therefore$ therefore

$$\forall x \in \mathbb{R} \quad \exists y \quad \ni \quad x+y=0$$

Power series: about $a=0$

$$c_0 + c_1 x + c_2 x^2 + \dots = \sum_{n=0}^{\infty} c_n x^n$$

about a

$$c_0 + c_1(x-a) + c_2(x-a)^2 + \dots = \sum_{n=0}^{\infty} c_n (x-a)^n$$

Ex

$$\sum_{n=1}^{\infty} \frac{(2x-1)^n}{5^n \sqrt{n}} = \frac{(2x-1)}{5} + \frac{(2x-1)^2}{25\sqrt{2}} + \dots$$

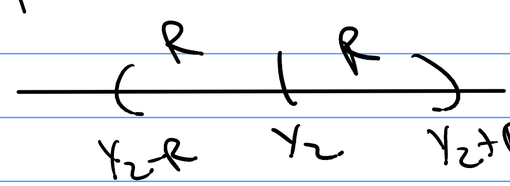
know

$$(2x-1)^n = \left(2\left(x-\tfrac{1}{2}\right)\right)^n = 2^n \left(x-\tfrac{1}{2}\right)^n$$

$$\sum_{n=1}^{\infty} \frac{2^n \left(x-\frac{1}{2}\right)^n}{5^n \sqrt{n}} = \sum_{n=1}^{\infty} \frac{2^n}{5^n \sqrt{n}} \left(x-\tfrac{1}{2}\right)^n$$

$$= \sum_{n=1}^{\infty} \underbrace{\boxed{\frac{(2/5)^n}{\sqrt{n}} \left(x-\tfrac{1}{2}\right)^n}}_{a_n}$$

power series about $a = 1/2$



ratio test:

$$\lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right| = L$$

$$a_{n+1} = \frac{(2/5)^{n+1}}{\sqrt{n+1}} (x - 1/2)^{n+1} \qquad a_n = \frac{(2/5)^n}{\sqrt{n}} (x - 1/2)^n$$

$$\lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right| = \lim_{n\to\infty} \frac{\frac{(2/5)^{n+1}}{\sqrt{n+1}} |x-1/2|^{n+1}}{\frac{(2/5)^n}{\sqrt{n}} |x-1/2|^n} = \lim_{n\to\infty} \frac{(2/5)^{n+1}}{(2/5)^n} \cdot \frac{\sqrt{n}}{\sqrt{n+1}} \cdot \frac{|x-1/2|^{n+1}}{|x-1/2|^n}$$

$$= \lim_{n\to\infty} \frac{2}{5} \cdot \frac{\sqrt{n}}{\sqrt{n+1}} \cdot |x - 1/2| = \boxed{\frac{2}{5} |x - 1/2| = L}$$

we found $L = \frac{2}{5} \left| x - \frac{1}{2} \right|$

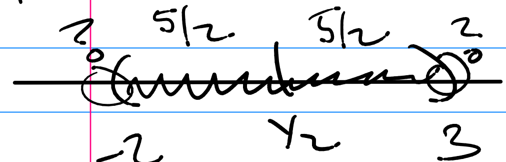
$\boxed{\text{conv}}$ $L < 1$ so $\frac{2}{5} \left| x - \frac{1}{2} \right| < 1$

$$\left| x - \frac{1}{2} \right| < \frac{5}{2} \rightarrow R = 5/2 \qquad \boxed{|x-a| < R}$$

$$-5/2 < x - \frac{1}{2} < 5/2$$

$$-2 < x < 3$$

Power series about $a = 1/2$



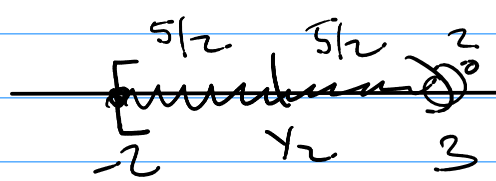
end pts: $L = 1$ have to study them separately

$\boxed{\text{case 1}}$ $x = -2$ then $\sum_{n=1}^{\infty} \frac{(2x-1)^n}{5^n \sqrt{n}} \rightarrow \sum_{n=1}^{\infty} \frac{(-5)^n}{(5)^n \sqrt{n}}$

$$\rightarrow \sum_{n=1}^{\infty} \frac{(-1)^n \cancel{5^n}}{\cancel{5^n} \sqrt{n}} = \sum_{n=1}^{\infty} (-1)^n \frac{1}{\sqrt{n}}$$

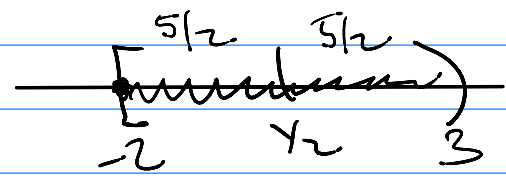
conv. alt. series @ $x = -2$

So..



$\boxed{\text{Case 2}}$ $x=3$ then $\sum_{n=1}^{\infty} \frac{(2x-1)^n}{5^n \sqrt{n}} \rightarrow \sum_{n=1}^{\infty} \frac{\cancel{5^n}}{\cancel{5^n} \sqrt{n}}$

$\rightarrow \sum_{n=1}^{\infty} \frac{1}{n^{1/2}}$ p-series, $p = 1/2$ $\boxed{SO}$ divergent

So ...  is the natural domain of

$$\sum_{n=1}^{\infty} \frac{(2x-1)^n}{5^n \sqrt{n}} = \frac{(2x-1)}{5} + \frac{(2x-1)^2}{25\sqrt{2}} + \cdots$$

power series about $x = 1/2$ & radius of conv $R = 5/2$

and interval of conv (domain) $[-2, 3)$

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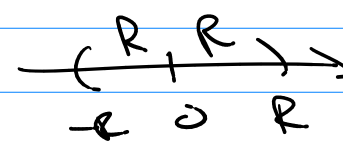
ex: $$\sum_{n=1}^{\infty} \frac{n!}{1\cdot 3\cdot 5 \cdots (2n-1)} x^n = \underset{n=1}{\frac{1}{1} x^1} + \underset{n=2}{\frac{1\cdot 2}{1\cdot 3} x^2} + \underset{n=3}{\frac{1\cdot 2\cdot 3}{1\cdot 3\cdot 5} x^3} + \ldots$$

know: power series about $a=0$  ?

$$\lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right| = L \qquad a_n = \frac{n!}{1\cdot 3\cdot 5 \cdots (2n-1)} x^n$$

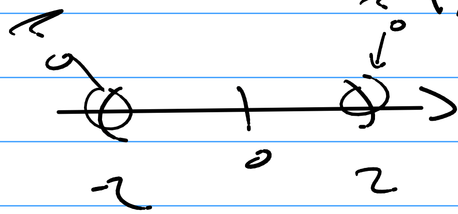
$$a_{n+1} = \frac{(n+1)!}{1\cdot 3\cdot 5 \cdots (2n-1)(2n+1)} x^{n+1}$$

$$\lim_{n\to\infty} \frac{\frac{(n+1)!}{1\cdot 3 \cdots (2n-1)(2n+1)} |x|^{n+1}}{\frac{n!}{1\cdot 3 \cdots (2n-1)} |x|^n} = \lim_{n\to\infty} \left| \frac{(n+1)}{(2n+1)} \right| |x| =$$

$$= \lim_{n\to\infty} \frac{1+1/n}{2+1/n}\,|x| = \frac{1}{2}|x| = L$$

So $\boxed{\text{conv}}$ $\quad L < 1 \to \frac{1}{2}|x| < 1$

$$\to |x| < 2 = R$$



$\boxed{\text{Case 1}}$ let $x = -2$ so $\displaystyle\sum_{n=1}^{\infty} \frac{n!}{1\cdot 3\cdots(2n-1)}(-2)^n$

$$= \sum_{n=1}^{\infty} (-1)^n \boxed{\frac{n!}{1\cdot 3\cdots(2n-1)}(2)^n}$$

$\to$ If this goes to zero

then conv.