Math 243

Q's $\frac{1}{(7+x)^2}$ as power series ...

$$(1+x)^p = 1 + px + \frac{p(p-1)}{2!}x^2 + \frac{p(p-1)(p-2)}{3!}x^3 + \cdots$$

$$\frac{1}{(7+x)^2} = \frac{1}{49}\left(1+x/7\right)^{-2} =$$

or

$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + \cdots \qquad |x|<1$$

$$\frac{d}{dx}\left[\frac{1}{1-x}\right] = \frac{1}{(1-x)^2} = \underline{1 + 2x + 3x^2 + 4x^3 + \cdots} \qquad |x|<1$$

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ex
$$\frac{9}{(6+x)^3} = \frac{9}{6^3(1+x/6)^3} = \frac{9}{6^3}(1+\boxed{x/6})^{-3\,p}$$

$$(1+\square)^p = 1 + p\square + \frac{p(p-1)}{2!}\square^2 + \frac{p(p-1)(p-2)}{3!}\square^3 + \cdots$$

$$\frac{9}{6^3}\left[1 + (-3)\left(\frac{x}{6}\right) + \frac{(-3)(-4)}{2!}\left(\frac{x}{6}\right)^2 + \frac{(-3)(-4)(-5)}{3!}\left(\frac{x}{6}\right)^3 + \cdots\right]$$

$$= \frac{9}{6^3}\sum_{n=0}^{\infty}\binom{-3}{n}\left(\frac{x}{6}\right)^n$$

☒ Applications of power series (Taylor / Maclaurin)

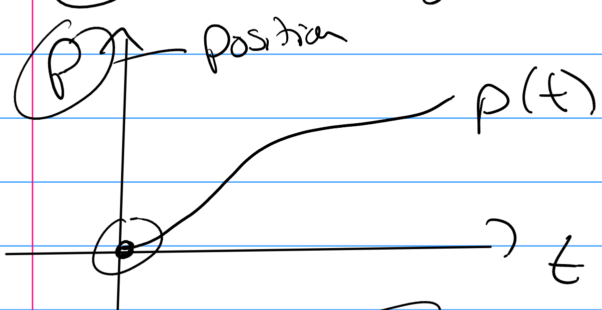
① (Math) $f(x) \approx$ polynomial of degree K.

@ $x=a$ $\quad f(x) \approx T_K$

$$f(x) \approx \underbrace{f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \cdots + \frac{f^{(K)}(a)}{K!}(x-a)^K}$$

② (physical models)

ex Car moving @ $20\ m/s$, accel @ $2\,(m/s)/s$



$p(t) \approx T_K$

$$p(t) \approx \boxed{p(0)} + p'(0)\,t + \frac{p''(0)}{2!}t^2 + \frac{p^{(3)}(0)}{3!}t^3 + \cdots + \frac{p^{(K)}(0)}{K!}t^K$$

position @ $t=0$ $\quad$ speed @ $t=0$ $\quad$ accel @ $t=0$ $\quad$ jerk @ $t=0$

$$p(t) \approx 0 + 20t + \frac{2}{2}t^2$$

$$p(t) \approx 20t + t^2$$

$p(1\,\text{sec}) \approx 21\ m$ $\rightarrow$ useful @ 60 sec?

$p(60) = 20 \cdot 60 + 60^2$

$= 4800\ m$

$v(60) = 20 + 2 \cdot 60 = 140\ m/s$

$$M = M_0 \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

at rest

v = velocity

c = speed of light

let $\frac{v}{c} = r$

"know" $0 \le v < c$ $\quad 0 \le r < 1$

$$M = M_0 \boxed{\frac{1}{\sqrt{1 - r^2}}}$$

$$\frac{1}{\sqrt{1-r^2}} = \left(1 + (-r^2)\right)^{-1/2} = 1 + (-1/2)(-r^2) + \frac{(-1/2)(-3/2)}{2!}(-r^2)^2 + \ldots$$

$$\frac{1}{\sqrt{1-r^2}} = 1 + \frac{1}{2}r^2 + \frac{3}{2^2 \cdot 2!}r^4 + \ldots$$

$$M = M_0\left(1 + \frac{1}{2}r^2 + \frac{3}{2^2 2!}r^4 + \ldots\right)$$

$0 \le r < 1$