Math 243

$\boxed{Q'S}$ $\displaystyle \lim_{x\to 0} \frac{1-\cos(2x)}{1+2x-e^{2x}} = \lim_{x\to 0}\frac{2\sin(2x)}{2-2e^{2x}} = \lim_{x\to 0}\frac{4\cos(2x)}{-4e^{2x}}$ type $\frac{0}{0}$ type $\frac{0}{0}$

$$= \frac{4}{-4} = \boxed{-1}$$

(VS) $\displaystyle \frac{1-\cos(2x)}{1+2x-e^{2x}} =$ Maclaurin Series ?

$$\cos(\square) = 1 - \frac{1}{2!}\square^2 + \frac{1}{4!}\square^4 - \frac{1}{6!}\square^6 + \dots \qquad R=\infty$$

$$e^{\square} = 1 + \square + \frac{1}{2!}\square^2 + \frac{1}{3!}\square^3 + \frac{1}{4!}\square^4 + \dots$$

$$1-\cos 2x = \frac{1}{2!}(2x)^2 - \frac{1}{4!}(2x)^4 + \frac{1}{6!}(2x)^6 - \dots$$

$$1+2x-e^{2x} = -\frac{1}{2!}(2x)^2 - \frac{1}{3!}(2x)^3 - \frac{1}{4!}(2x)^4 - \dots$$

$$\frac{1-\cos 2x}{1+2x-e^{2x}} = \frac{\frac{2^2}{2!}x^2 - \frac{2^4}{4!}x^4 + \frac{2^6}{6!}x^6 - \dots}{-\frac{2^2}{2!}x^2 - \frac{2^3}{3!}x^3 - \frac{2^4}{4!}x^4 - \dots}$$

$$= \frac{-1 + \frac{2^2}{4\cdot 3}x^2 - \frac{2^4}{6\cdot5\cdot4\cdot3}x^4 + \dots}{1 + \frac{2}{3}x + \frac{2^2}{4\cdot3}x^2 + \dots}$$

$$\lim_{x\to 0}\left(\frac{1-\cos 2x}{1+2x-e^{2x}}\right) = \lim_{x\to 0}\left[\frac{-1 + \frac{2^2}{4\cdot 3}x^2 - \frac{2^4}{6\cdot5\cdot4\cdot3}x^4 + \dots}{1 + \frac{2}{3}x + \frac{2^2}{4\cdot3}x^2 + \dots}\right]$$

$$= \boxed{-1}$$

Speed of light is a constant ... $= c$

$$\text{Energy} = \text{mass} \cdot c^2$$

<u>Kinetic Energy</u> (Newton) $K = \frac{1}{2} m v^2$

<u>Energy difference</u> $\begin{pmatrix}\text{Energy in} \\ \text{motion}\end{pmatrix} - \begin{pmatrix}\text{Energy} \\ \text{@ rest}\end{pmatrix}$

$$\boxed{K = mc^2 - m_0 c^2}$$

$m = \underline{(1-r^2)}^{-1/2} m_0 \qquad r = \frac{v}{c} \qquad 0 \le r < 1$

$$m = m_0 \left[1 + \frac{1}{2} r^2 + \frac{3}{2^2 2!} r^4 + \dots \right]$$

$$\underset{\text{in motion}}{\text{Energy}} = mc^2 = m_0 c^2 \left[1 + \frac{1}{2} r^2 + \frac{3}{2^2 2!} r^4 + \dots \right]$$

$$\underset{\text{@ rest}}{\text{Energy}} = m_0 c^2$$

$$K = \begin{pmatrix}\text{Energy} \\ \text{in motion}\end{pmatrix} - \begin{pmatrix}\text{Energy} \\ \text{@ rest}\end{pmatrix} = \frac{1}{2} m_0 c^2 r^2 + \boxed{\cancel{\frac{3}{2^2 2!} m_0 c^2 r^4 + \dots}} \to 0$$

in Newton's physics

$$r = \frac{\overset{10^2}{v}}{\underset{10^8}{c}} \qquad r \approx 10^{-6}$$

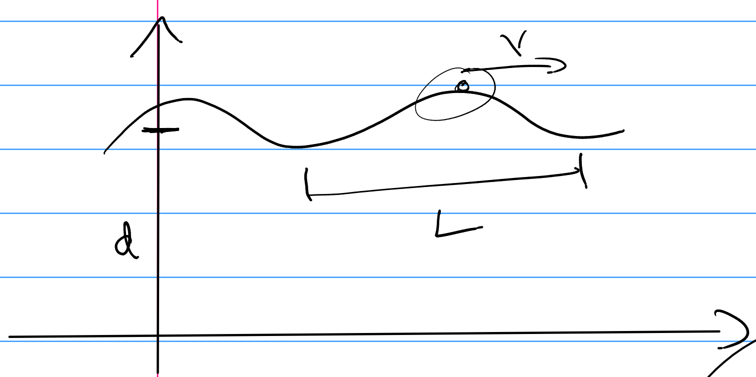
$\to$ small r $\quad K \approx \frac{1}{2} m_0 v^2$

(x, y, z, k), t

Time Dilation: $T = (1-r^2)^{-1/2} T_0$

Spacial Contraction $L = (1-r^2)^{1/2} L_0$

$$L = L_0\left(1 - \frac{1}{2} r^2 - \frac{1}{2^2 2!} r^4 - \dots\right)$$



$$v = \left(\frac{gL}{2\pi} \tanh\left(2\pi \frac{d}{L}\right)\right)^{1/2}$$

$\frac{d}{L}$

Depth << length
$\hookrightarrow \frac{d}{L} \sim 0$

Shore

depth >> length
$\hookrightarrow \frac{d}{L} \sim \infty$

deep ocean

let $\frac{d}{L} = r \rightarrow L = \frac{d}{r}$

so $$v = \left(\frac{gL}{2\pi} \tanh\left(2\pi \frac{d}{L}\right)\right)^{1/2}$$

$$v(r) = \left(\frac{gd}{2\pi r} \underbrace{\tanh(2\pi r)}\right)^{1/2}$$

$$\tanh(x) = \frac{\sinh x}{\cosh x} = \frac{e^x - e^{-x}}{e^x + e^{-x}}$$ or $$f(x) = f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \dots$$

Do it

$$\tanh(x) = x - \frac{1}{3}x^3 + \frac{2}{15}x^5 - \dots$$

$$\rightarrow \tanh(2\pi r) = \left[2\pi r - \frac{(2\pi)^3}{3} r^3 + \frac{2(2\pi)^5}{15} r^5 - \dots\right]$$

$$v = \left[\frac{gd}{2\pi r} \tanh(2\pi r)\right]^{1/2} = \left[gd - \frac{gd}{2\pi r} \frac{(2\pi)^3}{3} r^3 + \dots\right]^{1/2}$$

deep $r \to \infty$ $\lim_{r \to \infty} \tanh(2\pi r) = 1$

<u>shallow</u> $r \sim 0$ $\boxed{v \approx \sqrt{gd}}$