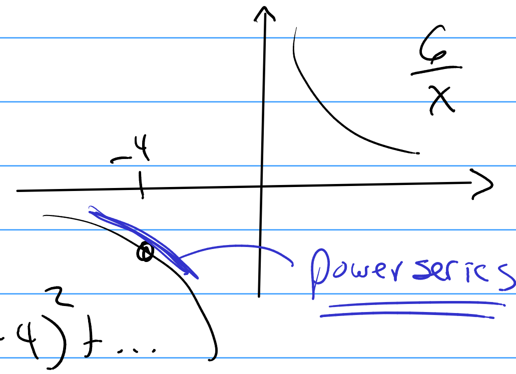
Math 243

Q15

$f(x) = \frac{6}{x} \qquad a = -4$

$$\frac{6}{x} = \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n$$



$$\frac{6}{x} = f(-4) + f'(-4)(x+4) + \frac{f^{(2)}(-4)}{2!}(x+4)^2 + \dots$$

$$f(x) = 6x^{-1} = +6 \cdot 0!\, x^{-1}$$
$$f'(x) = -6x^{-2} = -6 \cdot 1!\, x^{-2}$$
$$f^{(2)}(x) = 6 \cdot 2\, x^{-3} = +6 \cdot 2!\, x^{-3} \qquad @\ x = -4$$
$$f^{(3)}(x) = -6 \cdot 3 \cdot 2 \cdot x^{-4} = -6 \cdot 3!\, x^{-4}$$
$$f^{(4)}(x) = 6 \cdot 4 \cdot 3 \cdot 2 \cdot x^{-5} = +6 \cdot 4!\, x^{-5}$$

$$f^{(n)}(x) = (-1)^n\, 6 \cdot n! \cdot x^{-(n+1)}$$

@ $x = -4$

$$f^{(n)}(-4) = (-1)^n\, 6\, n!\, (-4)^{-(n+1)} = (-1)^n\, 6\, n!\, (-1)^{-n-1}\, 4^{-(n+1)}$$

$$= \frac{-6\, n!}{4^{n+1}}$$

$$\frac{6}{x} = \sum_{n=0}^{\infty} \frac{f^{(n)}(-4)}{n!}(x+4)^n = \sum_{n=1}^{\infty} \frac{-6}{4^n} \cdot (x+4)^{n-1}$$

$$\boxed{\frac{6}{x} = -\frac{3}{2} - \frac{6}{4^2}(x+4) - \frac{6}{4^3}(x+4)^2}$$

Up Next Today → Review probs for Exam 4

Monday Calc 3 (+) Linear Alg (+) Questions

Tuesday Diff Eq (+) Questions

Wednesday **Exam 4**

Thursday Review Probs for Final

→ Monday May 7^{th} @ 9am

Exam 4 11 probs @ 10pts, 100pts = 100%

Seq / Series

11.1 Seq $\{a_n\}$, $a_n = f(n)$ $n = 1, 2, \ldots$

1 prob limit of a sequence (may have parts)

1a $\lim_{n\to\infty} \frac{n}{2n+4} =$

1b $\lim_{n\to\infty} (\quad) \leftarrow$ type $\frac{\infty}{\infty}$ so move to x's and ...

11.2 Prove formula for $\sum_{n=0}^{\infty} ar^n = \frac{a}{1-r}$ if $|r|<1$

① $\lim_{k\to\infty} S_k = ?$

$S_1 = a$

$S_2 = a + ar$

$S_3 = a + ar + ar^2$

$\vdots$

$S_k = a + ar + ar^2 + \dots + ar^{k-1}$

Consider: $rS_k = r(a + ar + \dots + ar^{k-1})$

etc

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cf 11.2 ② find sum of a telescoping series.

(a) $\sum a_{n+1} - a_n$ middle cancels

$S_k = [\,] + \cancel{\dots} + [\,]$

$\lim_{k \to \infty} S_k \to ?$

Convergence of $\sum_{n=0}^{\infty} a_n$ (5 problems)

① use comparison test.

guess $\sum a_n$ is divergent

you do what? $a_n >$ something you know diverges

guess $\sum a_n$ is convergent

you do what? $a_n <$ something you know converges

② use limit comparison test.

③ Alt. Series. → conv? div?

→ if conv is a partial sum to the left or right of real ans? How accurate?

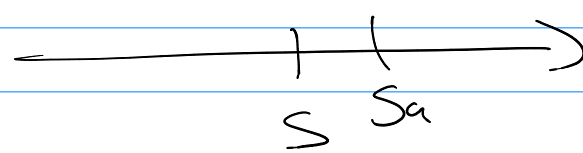
ex $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{n} = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \dots$

$\boxed{\text{Conv.}}$ b/c $\frac{1}{n}$ is dec. $\left(\frac{1}{n} > \frac{1}{n+1} \right)$

and $\lim_{n \to \infty} \frac{1}{n} = 0$

ex $S = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \dots$

$S \approx S_9 = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \frac{1}{6} + \frac{1}{7} - \frac{1}{8} + \frac{1}{9}$



error $< \frac{1}{10}$

④ abs conv. question

→ How? ① ratio/root test

② check $\sum |a_n|$ → no check $\sum a_n$ → Y / n

→ yes abs conv and conv.

⑤ absolute and/or conv.

11.9 Find power series for a function

2 probs

(1)
(2) $f(x) = c_0 + c_1x + c_2x^2 + \dots$

$$\frac{1}{1-\square} = 1 + \square + \square^2 + \dots \qquad |\square| < 1$$

$$e^{\square} = 1 + \square + \frac{1}{2!}\square^2 + \dots \qquad R = \infty$$

$$\frac{6}{4+x} = \frac{6}{4}\left[\frac{1}{1-(x/4)}\right]$$

11.10 Maclaurin Series (1 prob)

$$f(x) = f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \dots$$

$$R = ?$$