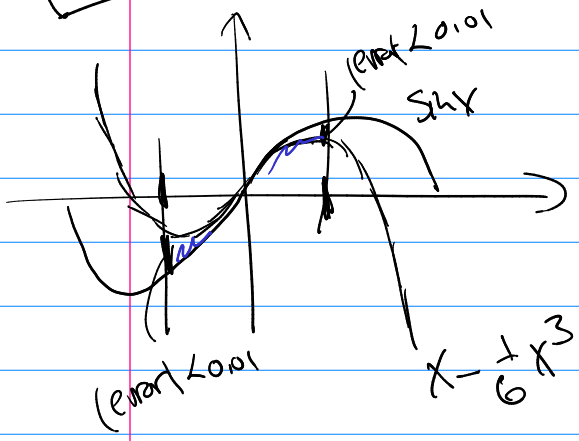
Math 243

Q's

$\sin(x) \approx x - \frac{1}{3!}x^3$



$|\text{error}| < 0.01$

$\text{error} \approx \frac{1}{5!}x^5$

$\left|\frac{1}{5!}x^5\right| < 0.01$

$|x|^5 < 1.2$

$|x| < (1.2)^{1/5}$

$\sum_{n=1}^{\infty} \frac{1}{n^2+n}$ use telescoping.

$\frac{1}{n^2+n} = \frac{1}{n(n+1)} = \frac{A}{n} + \frac{B}{n+1}$

So $1 = A(n+1) + Bn$ $\quad A = 1$, $B = -1$

$\sum_{n=1}^{\infty} \left(\frac{1}{n} - \frac{1}{n+1}\right)$

$S_k = \underbrace{\left(1 - \frac{1}{2}\right)}_{n=1} + \underbrace{\left(\frac{1}{2} - \frac{1}{3}\right)}_{n=2} + \underbrace{\left(\frac{1}{3} - \frac{1}{4}\right)}_{n=3} \cdots + \underbrace{\left(\frac{1}{k-1} - \frac{1}{k}\right)}_{n=k-1} + \underbrace{\left(\frac{1}{k} - \frac{1}{k+1}\right)}_{n=k}$

$S_k = 1 - \frac{1}{k+1}$ $\qquad \lim_{k\to\infty} 1 - \frac{1}{k+1} = \boxed{1}$

Comparison test

$$\sum_{n=} \frac{n^2-n}{\sqrt[3]{n^{10}+n^9}}$$

guess conv.

find $a_n <$ (bigger series that conv)

guess: $\frac{n^2-n}{\sqrt[3]{n^{10}+n^9}} \sim \frac{n^2}{\sqrt[3]{n^{10}}} = \frac{n^{6/3}}{n^{10/3}} = \frac{1}{n^{4/3}}$ (larger n) (so) guess conv.

b/c conv. p-series

comparison test

$$\frac{n^2-n}{\sqrt[3]{n^{10}+n^9}} < \frac{n^2-n+n}{\sqrt[3]{n^{10}+n^9}} = \frac{n^2}{\sqrt[3]{n^{10}+n^9}} < \frac{n^2}{\sqrt[3]{n^{10}}} = \frac{1}{n^{4/3}}$$

conv.

$\therefore \sum \frac{n^2-n}{\sqrt[3]{n^{10}+n^9}}$ is conv.

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limit comparison $\sum_{n=1}^{\infty} \frac{2n^2+3n}{\sqrt[3]{27n^{10}-n^9}}$

guess $\frac{2n^2+3n}{\sqrt[3]{27n^{10}-n^9}} \sim \frac{2n^2}{\sqrt[3]{27n^{10}}} = \frac{2}{3}\frac{1}{n^{4/3}}$ conv.

comparison $\frac{2n^2+3n}{\sqrt[3]{27n^{10}-n^9}} < \frac{2n^2+3n^2}{\sqrt[3]{27n^{10}-n^{10}}} = \frac{5n^2}{\sqrt[3]{26n^{10}}}$

limit comparison $\lim_{n\to\infty} \frac{\frac{2n^2+3n}{\sqrt[3]{27n^{10}-n^9}}}{\frac{1}{n^{4/3}}} = \lim_{n\to\infty} \frac{2n^{10/3}+3n^{7/3}}{\sqrt[3]{27n^{10}-n^9}}$

$$= \lim_{n\to\infty} \frac{2 + 3/n}{\sqrt[3]{27 - 4/n}} = \frac{2}{3}$$ pos. const. so they act alike → conv

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Calc 3 — ch12 - ch16

Calculus: limit, change, sum (infinite) on

functions: f ; domain → codomain

f ↑ Rule

Calc 1, 2 — $f: \mathbb{R} \to \mathbb{R}$ ex $y = 3x^2 + \sin 4x$

$1 \to 3 + \sin(1)$

ch12 — - Vectors (+) operations

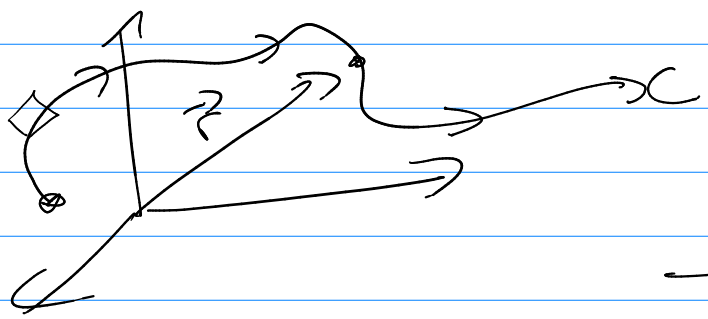
toys — - objects in 3D space: line, plane, cylinder, quartic

ch13 — Vector Functions

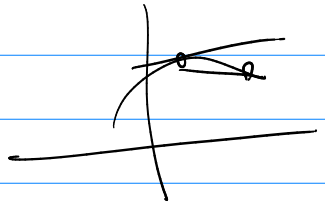
f: domain → codomain

$\mathbb{R}$ → Vectors.

$\vec{r}(t) = \langle t^2, 2t, 4 \rangle$



- limit
- change (deriv)
- sum (integral)

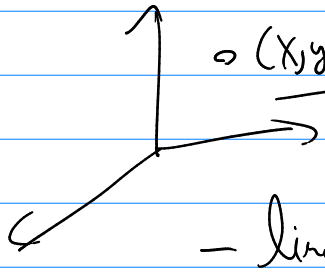


ch 14/15

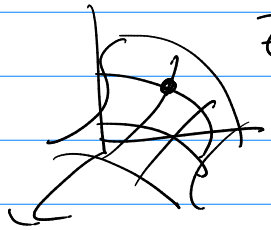
$f: \underline{\text{domain}} \to \mathbb{R}$

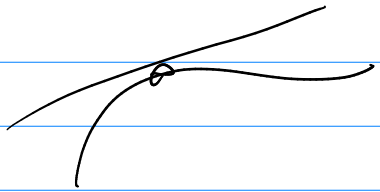
$\mathbb{R}^2$

$\mathbb{R}^2 \to \mathbb{R}$



- limit
- change partial deriv.





- double integral
  triple integrals