Math 243

Q's

Geo - sum

$$\sum_{n=0}^{\infty} ar^n = a + ar + ar^2 + \dots = S$$

to find $S = \lim_{k\to\infty} S_k$

$$S_k = a + ar + ar^2 + \dots + ar^k$$

let $rS_k = ar + ar^2 + \dots + ar^{k+1}$

So $S_k - rS_k = a - ar^{k+1}$

$$S_k(1-r) = a(1-r^{k+1})$$

$$S_k = a\,\frac{1-r^{k+1}}{1-r}$$

Now $\lim_{k\to\infty} S_k = \lim_{k\to\infty} a\left(\frac{1-r^{k+1}}{1-r}\right) = a\left(\frac{1-0}{1-r}\right)$

if $|r|<1$

So $\sum_{n=0}^{\infty} ar^n = \frac{a}{1-r}$ if $|r|<1$

Know: $\frac{1}{1-\square} = 1 + \square + \square^2 + \square^3 + \dots \quad |\square| < 1$

find power series of $\frac{1}{(3+x)^2}$

used: Need $\frac{1}{(\ )^n}$

start $\frac{1}{(\ )} \xrightarrow{\frac{d}{dx}} \frac{1}{(\ )^2} \xrightarrow{\frac{d}{dx}} \frac{1}{(\ )^3}$

$$\frac{1}{3+x} = \frac{1}{3}\left(\frac{1}{1-(-x/3)}\right) = \frac{1}{3}\left[1 - \frac{x}{3} + \frac{1}{3^2}x^2 - \frac{1}{3^3}x^3 + \dots\right]$$

when $\left|-x/3\right| < 1$ $\rightarrow$ $|x| < 3$

$$\frac{1}{3+x} = \frac{1}{3} - \frac{1}{3^2}x + \frac{1}{3^3}x^2 - \frac{1}{3^4}x^3 + \dots$$

$$\frac{d}{dx}\left[\frac{1}{3+x}\right] = \frac{d}{dx}\left[\frac{1}{3} - \frac{1}{3^2}x + \frac{1}{3^3}x^2 - \frac{1}{3^4}x^3 + \dots\right]$$

$$\frac{-1}{(3+x)^2} = 0 - \frac{1}{3^2} + \frac{2}{3^3}x - \frac{3}{3^4}x^2 + \frac{4}{3^5}x^3 - \dots$$

$$\boxed{\frac{1}{(3+x)^2} = \underset{n=0}{\frac{1}{3^2}} - \underset{n=1}{\frac{2}{3^3}x} + \underset{n=2}{\frac{3}{3^4}x^2} - \underset{n=3}{\frac{4}{3^5}x^3} + \dots \text{ on } |x|<3}$$

$$\boxed{\frac{1}{(3+x)^2} = \sum_{n=0}^{\infty} (-1)^n \frac{(n+1)}{3^{n+2}} x^n \quad \text{on} \quad |x|<3}$$

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(15) $$\frac{1}{1-\square} = \sum_{n=0}^{\infty} \square^n \quad \text{on} \quad |\square|<1$$

$$\frac{1}{3+x} = \frac{1}{3}\left(\frac{1}{1-(-x/3)}\right) = \frac{1}{3}\sum_{n=0}^{\infty}(-x/3)^n$$

$$\frac{1}{3+x} = \frac{1}{3}\sum_{n=0}^{\infty}(-1)^n \frac{1}{3^n}x^n \quad \text{on} \quad \left|\frac{-x}{3}\right|<1 \rightarrow |x|<3$$

$$\frac{1}{3+x} = \sum_{n=0}^{\infty}(-1)^n \frac{1}{3^{n+1}}x^n$$

So $\frac{d}{dx}\left[\frac{1}{3+x}\right] = \sum_{n=0}^{\infty} \frac{d}{dx}\left[(-1)^n \frac{1}{3^{n+1}} x^{n}\right]$

$$\frac{-1}{(3+x)^2} = \sum_{n=1}^{\infty} (-1)^n \frac{n}{3^{n+1}} x^{n-1}$$

$$\frac{1}{(3+x)^2} = \sum_{n=1}^{\infty} (-1)^{n+1} \frac{n}{3^{n+1}} x^{n-1}$$

$$\boxed{\frac{1}{(3+x)^2} = \sum_{n=0}^{\infty} (-1)^n \frac{n+1}{3^{n+2}} x^n}$$ on $|x|<3$

$R=3$

Maclaurin $f(x) = f(0) + f'(0)x + \frac{f^{(2)}(0)}{2!}x^2 + \frac{f^{(3)}(0)}{3!}x^3 + \ldots$

Find R by ratio test.

@ $x=0$

$f(x) = \cos(x^2)$ $\qquad f(0) = 1$

$f'(x) = -2x\sin(x^2)$ $\qquad f'(0) = 0$

$f^{(2)}(x) = -2\sin(x^2) - 4x^2\cos(x^2)$ $\qquad f^{(2)}(0) = 0$

$f^{(3)}(x) = -4x\cos(x^2) - [8x\cos(x^2) - 8x^3\sin(x^2)]$ $\qquad f^{(3)}(0) = 0$

$f^{(4)}(x) = -4\cos(x^2) + \square \; -8\cos(x^2) + \square + \square$ $\qquad f^{(4)}(0) = -12$

$\cos(x^2) = 1 + 0x + 0x^2 + 0x^3 + \frac{-12}{4!}x^4 + 0x^5 + 0x^6 + 0x^7 +$

$-\frac{1}{2!}$

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hence $\cos(x) = 1 - \frac{1}{2!}x^2 + \frac{1}{4!}x^4 - \frac{1}{6!}x^6 + \ldots$ $\quad R=\infty$

$\cos(x^2) = 1 - \frac{1}{2!}(x^2)^2 + \frac{1}{4!}(x^2)^4 - \frac{1}{6!}(x^2)^6 + \ldots$

$\boxed{\text{Diff Eqn's}}$

Solve consider College Algebra (Solve = find numbers that make eqn true)

$\boxed{3x-4=2(x-7)}$ $x^2-8=x(x+2)$ $\boxed{x^3+8=0}$

$\boxed{x=-10}$ $ab=0$

$a=0$ $b=0$

$\boxed{w^4-2w^2+3=0}$

$u^2-2u+3=0 \quad u=w^2$

Ordinary Diff Eqn's

equality

have $f(), f'(), f^{(2)}(), \ldots f^{(n)}()$

$$\frac{dT}{dt}=Tt \qquad \frac{dT}{dt}=\frac{T}{t}$$

Solve: $T(t)=\boxed{\text{expression}}$

ex $\boxed{y^{(1)}}+\boxed{y^{(1)}}+\sin(x)=x-2x\boxed{y}$

$y=f(x)=\boxed{\text{expression}}$ Find!

First Order Diff Eqn — subtypes → technique to learn

2nd order Diff Eqn

Systems of Diff. Eqn's

Preditor - Prey

$y(t)$ $x(t)$

$$\frac{dx}{dt} = \alpha x - \beta x y$$

$$\frac{dy}{dt} = \delta x y - \gamma y$$