

Math 243

Final Review

16 probs @ 10pts each

→ 160pts = 100%

Exam 1 → "take" 4 problems

ex $\int x^2 + \sin(x) \cos^3(x) dx$ [in an exam]

"take" is (1) exact copy?

or (2) small changes?

or (3) same concepts?

$\int x^{2/3} + \sin^3 x \cos x dx$

$\int \sec x \tan x + x \sqrt{3x^2+1} dx$

Exam 2 → 4 probs

Exam 3 → 4 probs

Exam 4 → 4 probs

EXAM 1 PROBLEMS ⁴ TAKEN FROM THIS EXAM

4 prob

1) Find an equation of the sphere with center (3,8,1) and passing through the point (2,6,4).

Spheres

info → eqn



2) Find the center and radius of the sphere: $x^2 + y^2 + z^2 - 2x + 4y + 3z = 10$

$$(x-h)^2 + (y-k)^2 + (z-l)^2 = r^2 \quad (h,k,l)$$

3) For $\mathbf{a} = \langle 1, 2, 3 \rangle$ and $\mathbf{b} = \langle -2, 1, 2 \rangle$ find ...

o

a) $|2\mathbf{a} - \mathbf{b}|$

b) $\mathbf{a} + (2\mathbf{b})$ and express your answer using the standard basis vectors.

operations: $\mathbf{a} \cdot \mathbf{b}$, $\mathbf{a} \times \mathbf{b}$

4) A W-lb weight hangs from two wires as shown. Find magnitude of the tensions, T1 and T2, in both wires. (with picture)

5) If you were given two vectors \mathbf{a} and \mathbf{b} , what would you do to determine if they were parallel?

o If they were perpendicular? (Explain your answers)

ops

6) A tow truck drags a car along a road. The chain makes an angle of 32° with the road and the tension in the chain is 1200 lbs. How much work is done by the truck if it is pulled 1 Kilometer?

7) Find a nonzero vector orthogonal to the plane through the points (1,0,1), (-1,1,3), and (4,2,2). Write your answer using the standard basis vectors. ops.

8) A bolt is tightened by applying a 10-lb force to the end of 20cm wrench at an angle of 61° . Find the magnitude of the torque about the center of the bolt. Draw a picture of the wrench and force so that the torque vector is into the bolt.

9) You study a bacterium and find that a single cell divides into two cells every 15 minutes. Given that you start with an initial population of 10 cells ...

a) Find the growth rate (use hours for the unit of time).

b) Find an expression for the number of cells after t hours.

10) When you take a cold drink from a refrigerator, its temperature is 34° F. After 18 minutes in a 72° F room the drink's temperature has increased to 45° F. What is its temperature if you wait an additional 14 minutes? When will its temperature be 67° F?

11) Perform the following ...

a) Prove that ~~$\frac{d}{dx}(\sec^{-1}(x)) = \frac{1}{x\sqrt{x^2-1}}$~~

b) Differentiate $f(t) = \ln(t) \sqrt{\arcsin(2t^2 + 3)}$

12) Find ...

a)

$$\int \frac{1+x}{1+x^2} + \frac{1}{x} + \sin(x) dx$$

b)

$$\int \frac{e^x}{\sqrt{1-e^{2x}}} dx$$

~~13) Prove that $\frac{d}{dx}(\cosh^{-1}(x)) = \frac{1}{\sqrt{x^2-1}}$~~

14) Find ...

a) the derivative of $x \tanh^{-1}(x) + \ln\sqrt{1-x^2}$

b) the integral

$$\int \frac{\cosh x}{\cosh^2 x - 1} dx$$

~~15) Find the limit~~

$$\lim_{x \rightarrow \infty} x^{(\ln 2)/(1+\ln(x))}$$

16) Find the limit

$$\lim_{x \rightarrow 0} \frac{\tanh(x)}{\tan(x)}$$

EXAM 2 PROBLEMS - 4 TAKEN FROM THIS EXAM

1) Use substitution and integration by parts to evaluate the given integral.

$$\int e^{\sin(t)} \sin(t) \cos(t) dt$$

? 2) Evaluate the given trigonometric integral.

$$\int \sin^7(x) \cos^5(x) dx$$

~~3) Use trigonometric substitution to evaluate the given integral.~~

$$\int x^3 \sqrt{1-x^2} dx$$

? 4) Use partial fractions to evaluate the given integral.

$$\int \frac{x^2 + x + 1}{x^3 + x} dx$$

? 5) Evaluate the given integral.

$$\int \frac{\tan^3(x)}{\cos^3(x)} dx$$

? 6) Evaluate the given integral.

$$\int e^x \sin(x) dx$$

~~7) Use substitution and the given table to evaluate the integral. (includes image of table)~~

$$\int \frac{1}{\sqrt{e^{2x} - 3}} dx$$

? 8) Setup the solutions for the left endpoint approximation and the simpson approximation to find the area of a pool that is 30 feet long and you took 11 width measurements of 0 ft, 5 ft, 10 ft, 12 ft, 15 ft, 12 ft, 10 ft, 10 ft, 12 ft, 6 ft, and 0 ft that were equally spaced along the length of the pool.

~~9) Find the arc length of the curve: $y = e^x$, $-\infty \leq x \leq 0$. (Hint: Use the table of integrals given early in the exam)~~

? 10) Setup, but do not integrate, an integrals to find the area of the surface and the volume generated by rotating the curve $y = e^{-2x}$, $0 \leq x \leq \infty$, about the x-axis.

~~11) A trough is filled with a liquid of density 10 lb/ft³. The ends of the trough are equilateral triangles with sides 10ft long and vertex at the bottom. Setup, but do not integrate, the integral to find the hydrostatic force on one end of the trough.~~

EXAM 3 PROBLEMS - 4 TAKEN FROM THIS EXAM

1) Sketch the parametric curve by eliminating the parameter to find a Cartesian equation.

$$x = t^2 - 2t, \quad y = t + 1, \quad 0 \leq t \leq 4$$

2) Find dy/dx and d^2y/dx^2 .

$$x = t^2 - 2t, \quad y = t + 1$$

3) Use the parametric equations of an ellipse, $x = 2\cos(t)$, $y = 3\sin(t)$, $0 \leq t \leq 2\pi$, to find the area that it encloses.

4) Find the exact length of the curve: $x = 1 + t^2$, $y = 1 + t^3$, $0 \leq t \leq 1$.

5) Plot the curves $r = 1 + \cos(\theta)$ and $r = 1 + \sin(\theta)$

6) Find the points on the curve $r = \cos(\theta)$ where the tangent is horizontal or vertical.

7) Find the area swept out between the curves $r = 1 + \sin(\theta)$ and $r = 1 + \cos(\theta)$ from $\theta = \pi/2$ and $\theta = \pi$.

8) Setup the integral to find the arc length of the polar curve formed by the area of the region inside $r = 4 \cos(\theta)$ and outside $r = 2$. Only SETUP the integral DO NOT SOLVE it.

9) Name and sketch the conic $9x^2 - 4y^2 - 72x + 8y + 176 = 0$.

10) Find the eccentricity, give an equation for the directrix, name and sketch the conic

$$r = 5 / (2 - 4 \cos(\theta)).$$

11) If you find the distance of a planet to the sun at it's perihelion to be r_p and at the aphelion to be r_a , then how would you find the planet's eccentricity e and semi-major axis a ?

EXAM 4 PROBLEMS - 4 TAKEN FROM THIS EXAM

1) Determine whether the sequence converges or diverges. If it converges, find the limit.

a) $a_n = \sqrt{3n+2} / \sqrt{n-4}$

b) $a_n = 3^n 7^{-n}$

c) $a_n = \ln(n)/\ln(2n)$

?
0 2) Show that the geometric series $a + ar + ar^2 + ar^3 + \dots = a/(1-r)$ when $|r| < 1$.

?
0 3) Find the sum of the telescoping series ...

$$\sum_{n=1}^{\infty} \frac{1}{n^2+n}$$

?
0 4) Use the comparison test to determine whether the series ...

$$\sum_{n=1}^{\infty} \frac{n+2}{n^2-1}$$

is convergent or divergent.

~~5) Use the limit comparison test to determine whether the series ...~~

$$\sum_{n=1}^{\infty} \frac{n^2+n}{\sqrt{1+n^7}}$$

is convergent or divergent.

?
0 6) Use the alternating series test to determine whether the alternating series ...

$$\sum_{n=1}^{\infty} \frac{(-9)^n}{n9^{n+1}}$$

is convergent or divergent.

7) Use the ratio test to determine whether the series ...

$$\sum_{n=1}^{\infty} \frac{(-9)^n}{n10^{n+1}}$$

is absolutely convergent.

8) Determine whether the series ...

$$\sum_{n=1}^{\infty} ne^{-n}$$

is absolutely convergent, conditionally convergent, or divergent

?
0
9) Find a power series representation with interval of convergence for the function ...

$$f(x) = \frac{x}{2x^2 + 1}$$

... by using the power series $1/(1-x) = 1 + x + x^2 + x^3 + \dots$ for $|x| < 1$

~~10) Find a power series representation with interval of convergence for the function ...~~

$$f(x) = \frac{x}{(4 + x^2)^2}$$

?
0
11) Find the Maclaurin series for $f(x) = \sin(x)$ and find its radius of convergence.