Math 243

(Q's) http://chaos.math.wichita.edu

(Q's) $\sum_{n=1}^{\infty} \frac{n+5}{\sqrt[3]{n^7+n^2}}$?

p-series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ $p>1 \to$ conv, $p \le 1 \to$ div

? Comparison or limit comparison

$\to$ compare? $\to \sum a_n$ acts like [...]

? Integral test. $\sum a_n$ to $\int_1^{\infty} f(x)\,dx$

$f(n) = a_n$

? Divergence $\sum_{n=1}^{\infty} a_n$ if $\lim_{n\to\infty} a_n \neq 0$ then $\sum a_n$ div.

$a_1 + a_2 + \ldots$

(So) $\sum_{n=1}^{\infty} \frac{n+5}{\sqrt[3]{n^7+n^2}}$ as n is large $\frac{n+5}{\sqrt[3]{n^7+n^2}} \sim \frac{n}{\sqrt[3]{n^7}} = \frac{1}{n^{4/3}}$

"like"

conv.

$\to$ compare to $\frac{1}{n^{4/3}}$ a conv. p-series

p-series test

(1) Comparison test or (2) limit comparison

$\frac{n+5}{\sqrt[3]{n^7+n^2}} < $ find something bigger that is conv.

$\lim_{n\to\infty} \frac{\frac{n+5}{\sqrt[3]{n^7+n^2}}}{\frac{1}{n^{4/3}}} = $ pos. const. if then they act alike

$\hookrightarrow \dfrac{n+5}{\sqrt[3]{n^7+n^2}} < \dfrac{n+5}{\sqrt[3]{n^7}} < \dfrac{n+n}{\sqrt[3]{n^7}} = \dfrac{2n}{\sqrt[3]{n^7}} = \dfrac{2}{n^{4/3}}$

if $n \geq 5$ — conv!

$\displaystyle\sum_{n=0}^{\infty} \frac{1+\sin n}{10^n}$ like? $\dfrac{1+\sin n}{10^n} \sim \dfrac{\text{const between 0 and 2}}{10^n}$

$\sim \dfrac{1}{10^n} = 1 \cdot \left(\dfrac{1}{10}\right)^n$

Conv. $a \cdot r^n$, $|r|<1$

Comparison:

$\dfrac{1+\sin n}{10^n} < \dfrac{3}{10^n} = 3\left(\dfrac{1}{10}\right)^n$ conv!

conv.

11.6 Absolute Conv. / Ratio Test / Root Test

Def If $\sum |a_n|$ converges, we say $\sum a_n$ absolutely converges.

Thm If $\boxed{\sum |a_n| \text{ converges}}$, then $\boxed{\sum a_n}$ also converges.

call $\sum a_n$ abs. conv.

Note: $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2}$ vs $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2}$

Notice: $\displaystyle\sum_{n=1}^{\infty} \left|\frac{(-1)^n}{n^2}\right| = \sum_{n=1}^{\infty} \frac{1}{n^2}$

So.. b/c $\displaystyle\sum_{n=1}^{\infty} \left|\frac{(-1)^n}{n^2}\right| = \sum_{n=1}^{\infty} \frac{1}{n^2}$ is a conv. p-series

we can say $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2}$ is abs. conv and also conv.

Consider $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \dots$

by alt. series test b/c $\frac{1}{n}$ is dec. and $\lim_{n\to\infty} \frac{1}{n} = 0$

so $\displaystyle\sum_{n=1}^{\infty} (-1)^{n+1}\frac{1}{n}$ conv.

Consider $\displaystyle\sum_{n=1}^{\infty} \left|\frac{(-1)^{n+1}}{n}\right| = \sum_{n=1}^{\infty} \frac{1}{n} = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \dots$

div. harmonic series

so we found $\displaystyle\sum_{n=1}^{\infty} a_n$ conv, but $\displaystyle\sum_{n=1}^{\infty} |a_n|$ div.

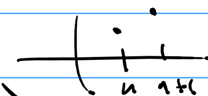
we say $\sum a_n$ is conditionally convergent.

Ratio and Root Tests

$\sum a_n$

$a_n = f(n)$

$a_{n+1} = f(n+1)$



Ratio Test

$$\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = L$$

① if $L < 1$ then $\sum a_n$ is abs. conv

② if $L > 1$ then $\sum a_n$ is div

③ $L = 1 \to$ test fails, do some other test

Root Test

$$\lim_{n\to\infty} \left(|a_n|\right)^{1/n} = L$$

① if $L < 1$ then $\sum a_n$ is abs. conv

② if $L > 1$ then $\sum a_n$ is div

③ if $L = 1 \Rightarrow$ test fails do something else

ex $\displaystyle\sum_{n=0}^{\infty} \frac{100^n}{n!} = 1 + \frac{100}{1} + \frac{10000}{2} + \frac{1{,}000{,}000}{6} + \dots$

Ratio test $\displaystyle\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty} \frac{\frac{100^{n+1}}{(n+1)!}}{\frac{100^n}{n!}} = \lim_{n\to\infty} \frac{100^{n+1}\, n!}{100^n (n+1)!}$

$$= \lim_{n\to\infty} 100 \cdot \frac{\cancel{n\cdot(n-1)\cdots(1)}}{(n+1)\,\cancel{n(n-1)\cdots(1)}} = \lim_{n\to\infty} 100 \cdot \frac{1}{n+1}^{\,0}$$

$$= 100 \cdot 0 = 0 = L$$

b/c $0<1$ $\qquad \sum \frac{100^n}{n!}$ abs. conv. $\to$ conv