Math 243

Taylor

$$f(x) = f(a) + f'(a)(x-a) + \frac{f^{(2)}(a)}{2!}(x-a)^2 + \cdots$$

$$= \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n$$

Maclaurin ($a=0$)

$$f(x) = f(0) + f'(0)x + \frac{f^{(2)}(0)}{2!}x^2 + \cdots, \quad R$$

$$= \sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!}x^n$$

Why?

① evaluate functions..

$$\sin(x) = 1 - \frac{1}{3!}x^3 + \frac{1}{5!}x^5 - \frac{1}{7!}x^7 + \cdots, \quad R = \infty$$

$$e^x = \exp(x) = 1 + x + \frac{1}{2!}x^2 + \frac{1}{3!}x^3 + \cdots, \quad R = \infty$$

$$\mathrm{erf}(x) = \frac{2}{\sqrt{\pi}}\left(x - \frac{1}{3 \cdot 1!}x^3 + \frac{1}{5 \cdot 2!}x^5 - \frac{1}{7 \cdot 3!}x^7 + \cdots\right), \quad R = \infty$$

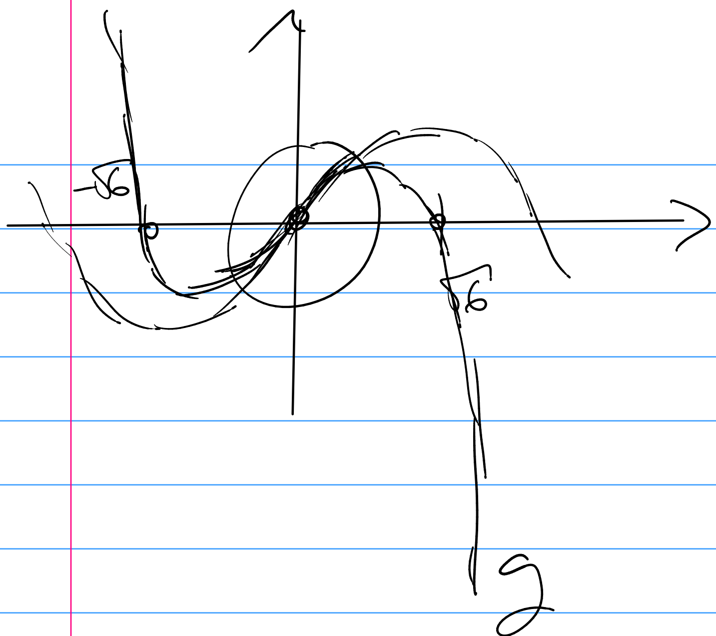
② approximate functions with polynomials.

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n$$

$f(x) \approx T_K$ — K^{th} power Taylor polynomial

$$\sin(x) \approx 1 - \frac{1}{3!}x^3 + \frac{1}{5!}x^5 + 0x^6 \qquad T_5 = T_6$$

so 6^{th} degree polynomial



$\sin x = 4$

$x - \frac{1}{3!}x^3 = g$

$x - \frac{1}{6}x^3$

$\frac{1}{6}x(6 - x^2) = 0$

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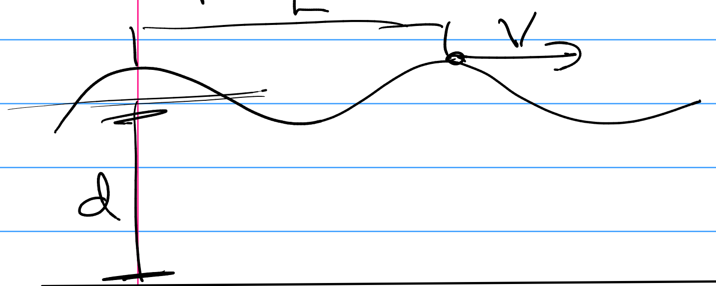
(3) physical model into polynomial approximate.



ocean wave

$$v = \left[\frac{gL}{2\pi}\tanh\left(2\pi\frac{d}{L}\right)\right]^{1/2}$$

$x \propto \sqrt{L}$

deep ocean $\frac{d}{L} \to \infty$ $(d \to \infty)$

shallow $\frac{d}{L} \to 0$

$$\Rightarrow \lim_{d\to\infty} \sqrt{\frac{gL}{2\pi}\cancel{\tanh\left(2\pi\frac{d}{L}\right)}} = \sqrt{\frac{gL}{2\pi}}$$

shallow $d \to 0$ $\tanh(\ )$?

near $d=0$ so.. use Maclaurin.

$$\tanh(x) = \tanh(0) + \frac{d}{dx}\left[\tanh(x)\right]\Big|_{x=0} x + \ldots$$

$$= 0 + \boxed{c_1} x + c_2 x^2 + \ldots$$

$$(10^{-4})^2 = 10^{-8}$$

$$\lim_{d \to 0} \sqrt{\frac{gL}{2\pi}\left[2\pi\left(\frac{d}{L}\right) + \cancel{(\ )d^2} + \cancel{(\ )d^3} + \ldots\right]}$$

$$\approx \boxed{\sqrt{gd}}$$

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ex

$$T = \frac{1}{(1-r^2)^{1/2}} T_0$$

$$L = (1-r^2)^{1/2} L_0$$

$$M = \frac{1}{(1-r^2)^{1/2}} M_0$$

v = velocity

c = speed of light

$$r = \frac{v}{c}$$

$$0 \le r < 1$$

$$K = \frac{1}{2} m v^2$$

Energy $= mc^2$

Kinetic Energy $=$ (Energy in motion) $-$ (Energy a rest)

$$mc^2 \qquad m_0 c^2$$

$$M = \frac{1}{\sqrt{1-\beta^2}} M_0$$

$$\text{Kinetic Energy} = \boxed{\frac{1}{\sqrt{1-\beta^2}}} M_0 c^2 - M_0 c^2$$

$$(1-\beta^2)^{-1/2} = \boxed{\left(1 + \frac{1}{2}\beta^2 + \frac{3}{2^2 2!}\beta^4 + \dots\right)}$$

$$K = \left(\cancel{M_0 c^2} + \frac{1}{2}\beta^2 M_0 c^2 + \dots\right) - \cancel{M_0 c^2}$$

$$K = \frac{1}{2}\beta^2 M_0 c^2 + \frac{3}{2^2 2!}\beta^4 M_0 c^2 + \dots$$

$$\beta^2 = \frac{v^2}{c^2}$$

$$K = \frac{1}{2} M_0 v^2 + \frac{3}{2^2 2!}\beta^4 M_0 c^2 + \dots$$

Exam 5

11st: Exam 1, Exam 2, Exam 3, Exam 4, Exam 5 (ch 11), Exam 6 (all chapters) + Final

but top 5 only go to grade

Monday   11 problems @ 10 pts   100pts = 100%

$\boxed{11.1}$ Seq   (1 prob)   May have parts.

$$\lim_{n\to\infty} (\text{seq}) = ?$$

$$\lim_{n\to\infty} \left(\frac{2n^3}{\sqrt{n^7+1}} \right)$$

$\boxed{11.2}$ 2 prob

① prove $\displaystyle\sum_{n=0}^{\infty} ar^n = \frac{a}{1-r}$, $|r|<1$

$$S_k = a + ar + ar^2 + \dots + ar^k$$

$$\lim_{k\to\infty} S_k = \dots = \frac{a}{1-r} \text{ if } |r|<1$$

② evaluate a telescoping series.

ex $\displaystyle\sum_{n=2}^{\infty} \frac{2}{n^2-1} = \sum_{n=2}^{\infty} \left[\frac{1}{n-1} - \frac{1}{n+1} \right]$

b/c $\displaystyle\frac{2}{n^2-1} = \frac{1}{n-1} - \frac{1}{n+1}$

$$S_k = \left[1 - \frac{1}{3}\right] + \left[\frac{1}{2} - \frac{1}{4}\right] + \left[\frac{1}{3} - \frac{1}{5}\right] + \dots + \left[\frac{1}{k-1} - \frac{1}{k+1}\right]$$

11.4 to 11.7 Convergent Series

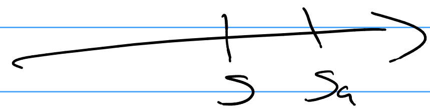
① use comparison test

② use limit comparison test

③ alt. series

$$S_9 = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \dots + \frac{1}{9}$$

$$S_9 \approx S \qquad \text{error} < \frac{1}{10}$$



④ is $\sum a_n$ abs. conv?

⑤ is $\sum a_n$ abs conv? cond conv? div?

Power Series

11.9 ① find power series knowing $\frac{1}{1-x}$ e^x

ex $\boxed{\ln(1+x) =} \int \frac{1}{1-(-x)}\,dx$

② Find Maclaurin series — $f(x) = f(0) + f'(0)x + \dots$

③ find a Taylor series — $f(x) = f(a) + f'(a)(x-a) + \dots$

$R = ?$