9.6 Contines

Hasse Diagrams

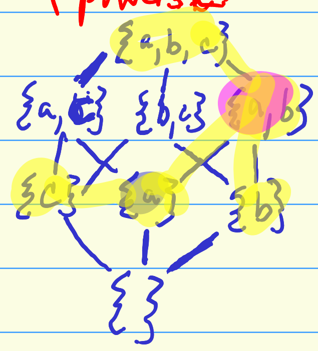
Posets are:
- reflexive
- antisym.
- trans.

Ex:

Let's draw the Hasse Diagram for $(P(\{a,b,c\}), \subseteq)$

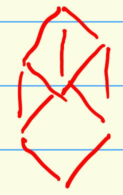
↑ powerset ← Relation

$\{\{\}, \{a\}, \{b\}, \{c\}, \{a,b\}, \{b,c\}, \{a,c\}, \{a,b,c\}\}$



$\{\} \subseteq \{a\}$

so $\{\}$ and $\{a\}$ are related



Cube

Huh...

Related Question: (a) What is/are the maximal & min. elements?

Max: $\{a,b,c\}$ min: $\{\}$.

(b) What is the greatest lower bound of $\{\{a\}, \{a,b\}, \{a,b,c\}\}$?

$\{a\}$

(c) What about the least upper bound of $\{\{a\}, \{a,b\}, \{b\}\}$

$\{a,b\}$.

Review

Q: Consider the relation R on the set of ordered pairs of integers where $((a,b),(c,d)) \in R$ if $ad = bc$. Show this rel'n is an equivalence, and write the logical definition of each prop. as you consider it.

$\boxed{(a,b) R (c,d)}$

Ex: $((1,2),(3,6)) \in R$

Since: $1 \cdot 6 = 2 \cdot 3$

$\frac{1}{2} = \frac{3}{6}$

(a) Reflexivity: $\forall a \ (a,a) \in R$

In this case, is $((a,b),(a,b)) \in R$?

Yes: $ab = ba$ ✓

(b) Symmetry $\forall a \forall b \ [(a,b) \in R \to (b,a) \in R]$

In this case, ~~consider~~ if $(\underbrace{(a,b)}_{\text{"a"}}, \underbrace{(c,d)}_{\text{"b"}}) \in R$,

then $ad = bc$.

$\Leftrightarrow be = ad$

$\Leftrightarrow cb = da$

Hence $(\underbrace{(c,d)}_{\text{"b"}}, \underbrace{(a,b)}_{\text{"a"}}) \in R$

goal $((c,d),(a,b)) \in R$

- $cb = da$

(c) Transitivity: $\forall a \forall b \forall c \ ([(a,b) \in R \wedge (b,c) \in R] \to (a,c) \in R)$

If: $((a,b),(c,d)) \in R$ and $((c,d),(e,f)) \in R$

$ad = bc$ and $cf = de$

Goal $((a,b),(e,f)) \in R$

$af = be$

$d = \frac{bc}{a}$ $\quad$ $cf = \left(\frac{bc}{a}\right)e$

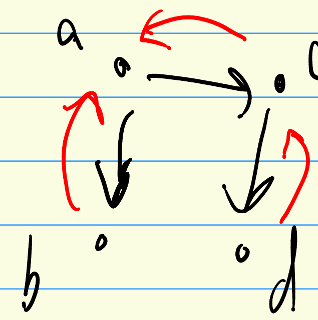
$cf = \frac{bce}{a}$

$af = be$

Therefore $((a,b),(e,f)) \in R$. ☺!!

So it's an equivalence!

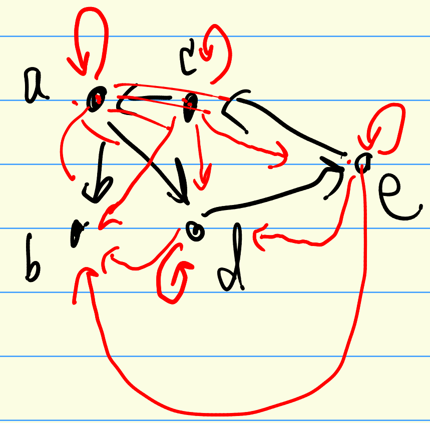
$\forall x \forall y [(x,y) \in R \rightarrow (y,x) \in R]$



$M_R = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$ is now symmetric

Trans closure

a lot (ew)



Smaller:

$M_R = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 \end{bmatrix}$

$M_R^{[2]} = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 \end{bmatrix} \odot \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 \end{bmatrix}$

$= \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 \end{bmatrix}$

a path of length 2 from a to c

$c \rightarrow a \rightarrow c$

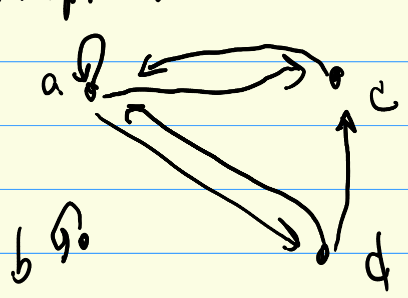
$b \rightarrow b \rightarrow b$

$M_R^{[3]} = M_R^{[2]} \odot M_R$

or $M_R \odot M_R^{[2]}$

Only b/c it's same matrix

Graph: of R



Warshall's Algorithm:

$M_R = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 \end{bmatrix}$

for k in 1 to n
  for i 1 to n
    for j 1 to n
      $w_{ij} = w_{ij} \vee [w_{ik} \wedge w_{kj}]$
  ;
 ;
return $[w_{ij}]$

$k = 1$

$k = 2$

$w_{i1} \wedge w_{1j}$

$w_{11} \wedge w_{11} \rightsquigarrow w_{11}$

$w_{11} \wedge w_{13} \rightsquigarrow w_{13}$

$w_{11} \wedge w_{14} \rightsquigarrow w_{14}$

$w_{31} \wedge w_{11} \rightsquigarrow w_{31}$ $w_{13} \rightsquigarrow w_{33}$ $w_{14} \rightsquigarrow w_{34}$

$w_{41} \wedge w_{11}$ w_{13} w_{14}