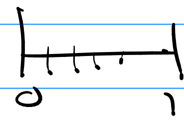
Math 322

Syllabus (chaos.math.wichita.edu)

$\mathbb{R}$ from 0 to 1 $\qquad r = 0.d_1d_2d_3d_4\ldots$

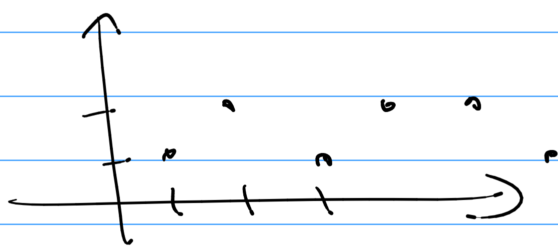
$f_r(n) = d_n$



ex $\quad r = 0.121221\,2221\,2222\ldots$

$f_r(n) = d_n \qquad f_r(3) = 1 \qquad (3,1) \in f_r$

$3 \mapsto 1$



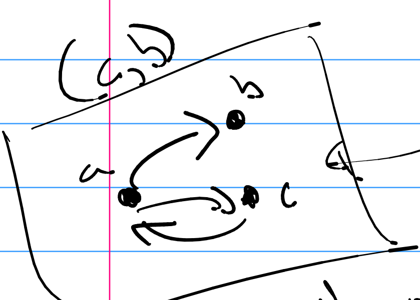
Discrete 2

$(a_1, a_2, \ldots, a_n)$

Sets $\rightarrow$ (Sets of ordered n-tuples) n-ary Relations

$\boxed{\text{set of ordered pairs}}$ Relation (new toy!)

Visualize



graph theory (new toys!)

special graph

$\boxed{\text{tree}}$

Pause Boolean $\boxed{\text{Algebra}}$ set = $\{O_1, O_2\}$

operations: unary
binary$_1$
binary$_2$

Laws

End human language (+) graphs

$\rightarrow$ $\boxed{\text{Model computation}}$

Relations relationship between n-sets.

(Mark, tie, LG Phone, chair #1) 4-tuple

People clothes phones chairs

Set People X clothes X phones X chairs

$= \{(a, b, c, d) \mid a \in \text{People} \wedge \ldots \text{etc}\}$ $\boxed{\text{Set builder}}$

$\boxed{\text{n-ary relation}}$ Subset of $A_1 \times A_2 \times \ldots \times A_n$

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n-sets is hard.

Step 1 only use two sets $A_1 \times A_2$

binary relation or just relation

R is a relation is a subset of $A_1 \times A_2$

$$R = \{ (a,b) \mid \underbrace{P(a,b)}_{\text{propositional function of } A, B} \}$$

ex $R = \{ (a,b) \mid a \leq b \}$

$A = \{1, 2, 3, 4\}$ $B = \{2, 3\}$

$|A \times B| = |A||B| = 8$ ordered pairs

$$A \times B = \{ (1,2), (1,3), (2,2), (2,3), (3,2), (3,3), (4,2), (4,3) \}$$

$$R = \{ (1,2), (1,3), (2,2), (2,3), (3,3) \}$$

Note: any relation is a subset of $A \times B$

how many?: $|P(A \times B)| = 2^8 = 256$ ← possible relations

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Notation: R is a subset of $A \times B$

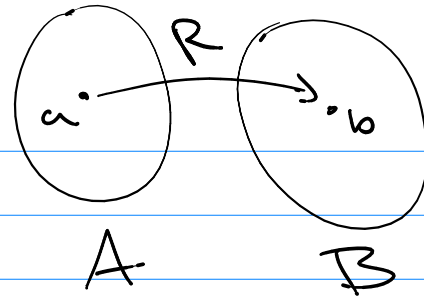
$R : A \to B$

domain codomain

$(a,b) \in R$ use aRb

$(a,b) \notin R$ use $a \not R b$

Visualize: aRb



$a \longrightarrow b$

Table

(a₂, b₁)

R	b_1	b_2	B ...	b_n
a_1				
A a_2	x			
$\vdots$				
a_n				

Zero-one matrix

$$R = \begin{bmatrix} 0 & 0 & \cdots & 1 \\ 0 & 1 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \end{bmatrix} \quad (a_2, b_2)$$

Note: Special relations?

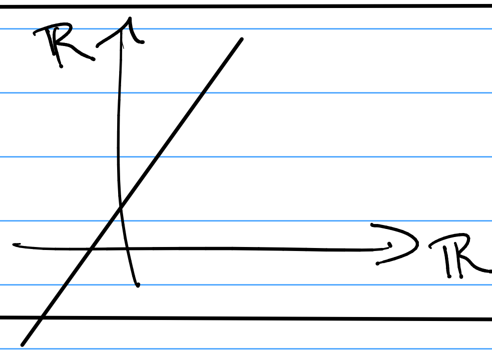
(i) Function: $f: A \to B$ $\quad f(a) = b$

relation where **every** $a \in A$ goes to **exactly one** $b \in B$.

Note: $f(x) = 2x + 1$

ex

$f: \mathbb{R} \to \mathbb{R}$



Def R is a relation on set A means $R: A \to A$ (or subset of $A \times A$)

ex $A = \{2, 3, 4, \ldots, 10\}$

$R = \{(a,b) \mid a \mid b\}$

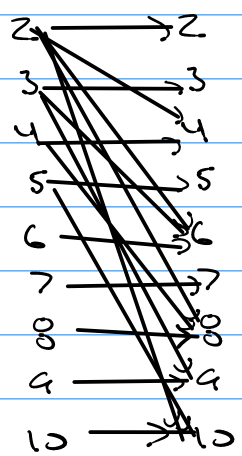
such that   divides

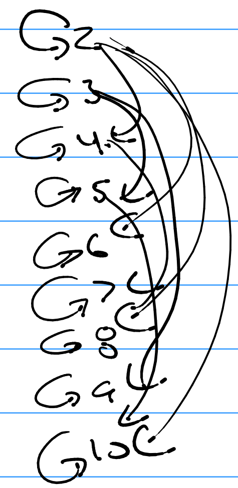
$a \mid b$ means $a \cdot k = b$

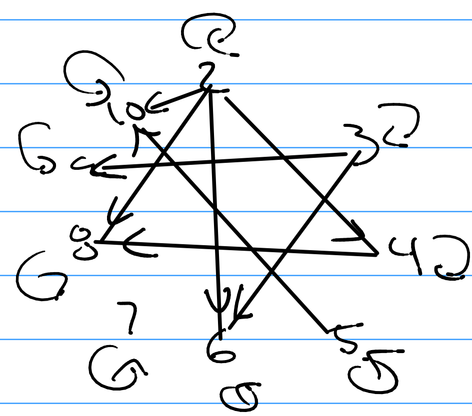
$\boxed{R} = \{(2,2), (2,4), (2,6), (2,8), (2,10),$
$(3,3), (3,6), (3,9), (4,4), (4,8)$
$(5,5), (5,10), (6,6), (7,7), (8,8), (9,9), (10,10)\}$

$\boxed{2R6}$ b/c $2 \mid 6$ b/c $(2,6) \in R$

$3 \not R\, 2$ b/c $3 \nmid 2$ b/c $(3,2) \notin R$







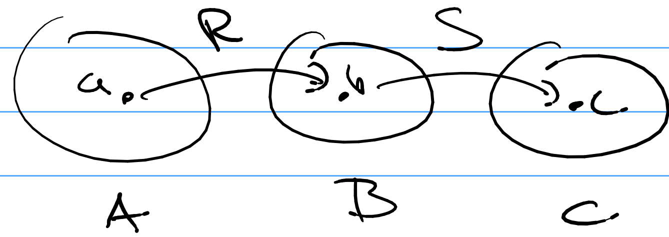
$$\begin{array}{c|ccccccccc}
 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \\
\hline
2 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \\
3 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\
4 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\
5 & & & & & & & & & \\
6 & & & & & \vdots & & & & \\
7 & & & & & & & & & \\
\vdots & & & & & & & & & \\
10 & & & & & & & & &
\end{array}$$

Operations?

(1) Set ops: $R_1 \cup R_2$, $R_1 \cap R_2$, $R_1 - R_2$, $R_1 \oplus R_2$, ..

exclusive or

(2) Composition:



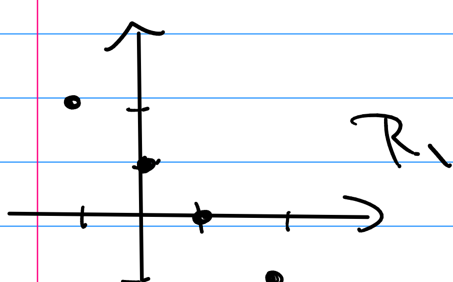
$$S \circ R = \{(a,c) \mid \exists b \;\; aRb \wedge bSc\}$$

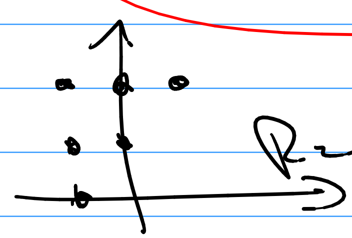
$$S(R(a))$$

$R_1 = \{(a,b) \mid a+b=1\}$ $\quad A = \{-1,0,1,2\}$

$R_2 = \{(a,b) \mid a<b\}$

$$R_1 \cup R_2 = \{(a,b) \mid (a+b=1) \vee (a<b)\}$$





single proposition?