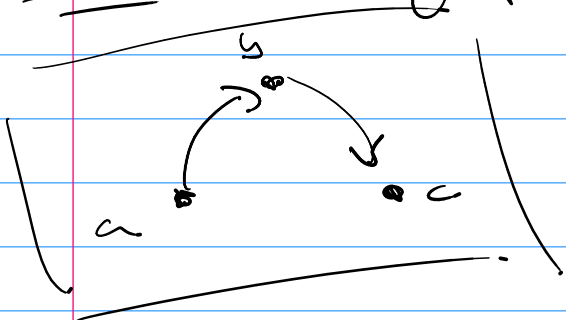
Math 322

Relations: → Digraph



R on set $A = \{a, b, c\}$

$R = \{(a,b), (b,c)\}$

ignore relations .. this alone seems useful!

New toys: Graphs

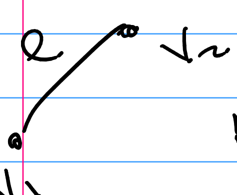
Def $G = (V, E)$ is a graph. Consist of two sets ...

① non-empty set V of vertices

② set V of edges

Subtypes ① Undirected Graph

- edges are unordered pairs of vertices $e = \{v_1, v_2\}$



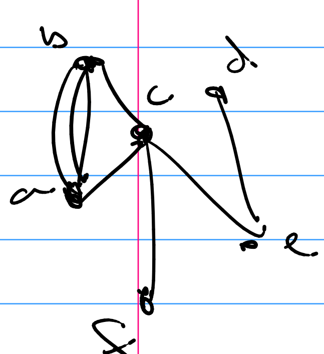
terms:

a) v_1, v_2 are endpoints of e.

b) e connects v_1, v_2

c) v_1, v_2 are adjacent or neighbors

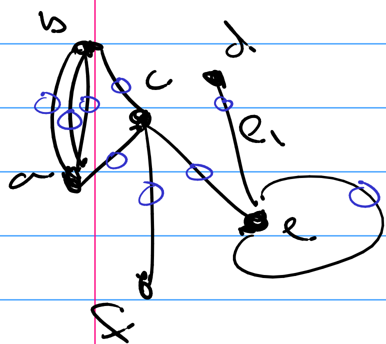
d) set of all neighbors to v is its neighborhood $N(v)$



$N(c) = \{a, b, f, e\}$

e) e is incident with v_1, v_2

f) degree of v, $\deg(v)$, is number of edges incident with v, loops count as 2.



	deg
a	4
b	4
c	4
d	1
e	4
f	1

sum = 18

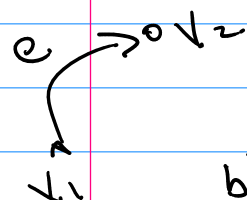
$|E| = 9$

Thm handshaking thm

$$\sum_{v \in V} \deg(v) = \boxed{2|E|}$$

Subtype **Directed Graphs**

edges are ordered pairs of vertices $e = (v_1, v_2)$



Terms

a) e starts @ v_1 (initial), e ends @ v_2 (terminal)

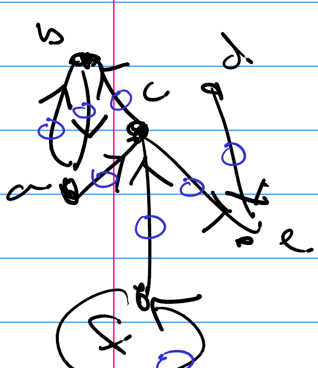
b) v_1 is adjacent to v_2, v_2 is adj. from v_1

c) In degree of v is # of edges with v as terminal

$\deg^-(v)$

Out degree of v is # of edges with v as initial

$\deg^+(v)$



	$\deg^-$	$\deg^+$
a	1	2
b	2	1
c	2	2
d	0	1
e	2	0
f	1	2

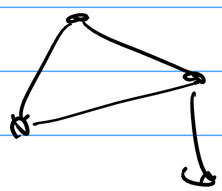
Thm $\sum_v \deg^-(v) = \sum_v \deg^+(v) = |E|$

	Undirected Graphs		Directed Graphs
Simple:	no mult. edges, no loops	Simple directed	no mult. edges, no loops
Multigraph	mult. edges are OK, no loops	directed **Multigraph**	mult. edges are OK, loops are OK
pseudograph	mult. edge are OK, loops are OK		

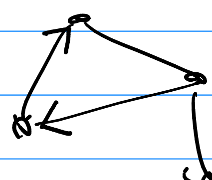
have both undirected and directed edges

→ $\boxed{\text{Mixed graph}}$

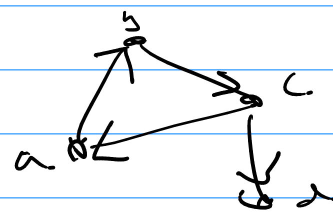
ex



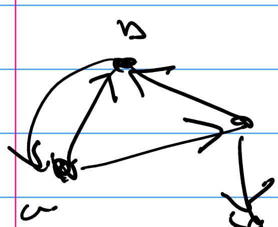
Simple



Mixed



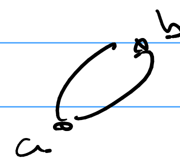
Simple directed



Simple directed

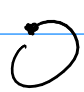
Note

Multiple edge:



{a,b}
{a,b}

(a,b)
(a,b)
(a,b)

 pseudograph

Why? Applications

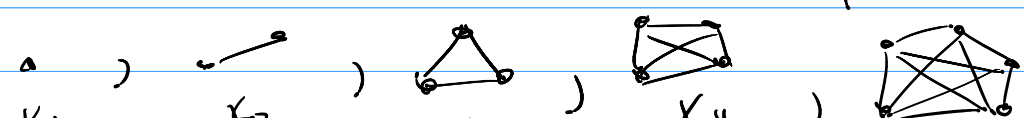
10.1

- logic flow
- transportation network
- Information networks
- call graphs
- Influence graphs

etc.

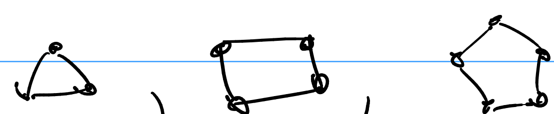
Special Graphs

Complete: K_n $(n \geq 1)$ every vertex connects to every other vertex.



K_1, K_2, K_3, K_4, K_5, ...

Cycle: C_n $(n \geq 3)$

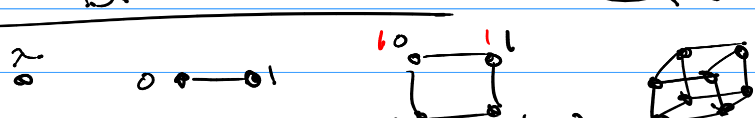


C_3, C_4, C_5, ...

Wheel W_n $(n \geq 3)$ $W_n = C_n \cup$ one axle vertex

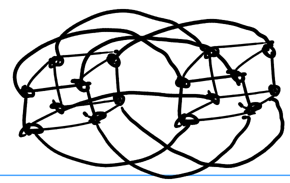
W_3, W_4, ...

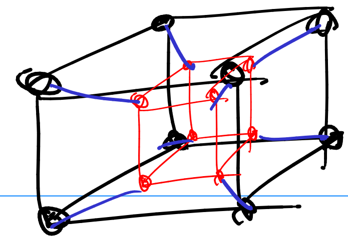
n^{th} Dimensional Cube Q_n



Q_0, Q_1, Q_2, Q_3, ...

Q_4





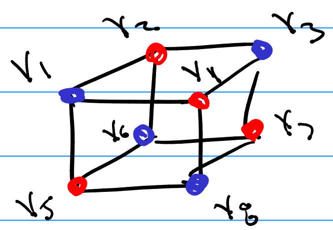
Q_4

Bipartite Graphs, Complete bipartite Graphs

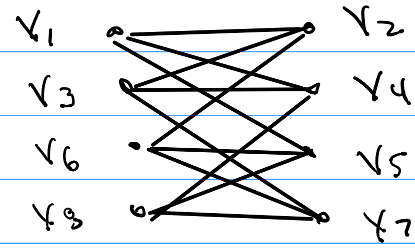
(check th$^{\underline{m}}$: Coloring th$^{\underline{m}}$)

G is a a simple graph and V can be partitioned into two sets such that edges only connect between the two sets.

ex

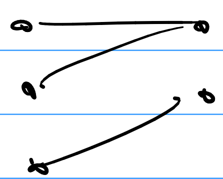


Q_3

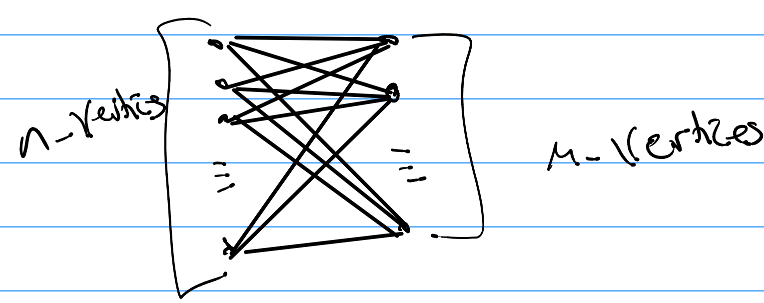


Q_3

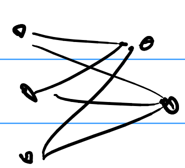
ex



ex Complete bipartite $K_{n,m}$



$K_{3,2}$



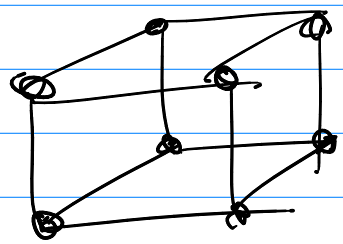
Operations

① <u>Subgraphs</u> H is a subgraph of G

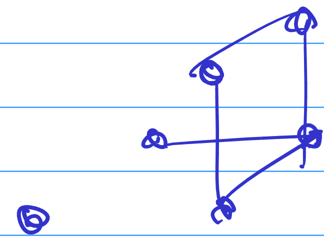
$H = (V_H, E_H)$ $G = (V_G, E_G)$

$V_H \subseteq V_G$, $E_H \subseteq E_G$

if $H \neq G$, proper subgraph



G



H

H is a subgraph of G