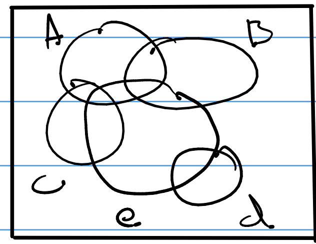
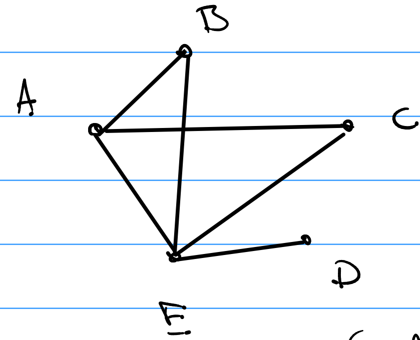
Math 322

Q's 10.1 #13

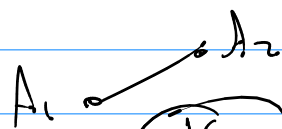




Intersection Graph

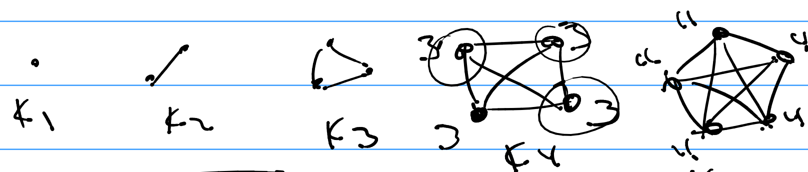
#13 c) $A_1 = \{x \mid x < 0\}$

$A_2 = \{x \mid -1 < x < 0\}$



etc

10.2 #35 K_n



Complete

$$\boxed{|V| = n}$$

$|E| = 6$ $\quad$ $|E| = 10$

$$\sum_{v \in V} \deg(v) = 2|E|$$

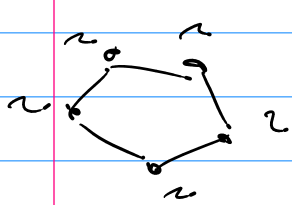
$$\deg(v) = n-1$$

$$|E| = \frac{1}{2} \sum_v \deg(v)$$

$$\boxed{|E| = \frac{n(n-1)}{2}}$$

C_n $\quad$ $|V| = n$ $\quad$ $|E| = \frac{1}{2}(2 \cdot n) = n$

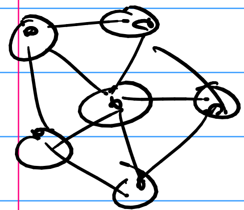
$\deg(v) = 2$



C_5

C_n + 1 vertex

W_n $\quad |V| = \boxed{n} + \boxed{1}$



$|E| = n + n = \boxed{2n}$

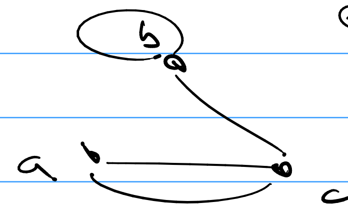
$\frac{1}{2}\left[3 \cdot n + n\right]$

W5

Representing Graphs

Undirected graph $\quad G = (V, E)$ $\quad$ Ex $V = \{a, b, c\}$

$E = \{ \{a,c\}^{e_1}, \{b,b\}, \{b,c\}, \{c,a\}_{e_2} \}$



① Adjacency lists

V	adj. with
a	c, c
b	b, c
c	a, a, b

② **Adj. Matrix** $\quad A_G = [a_{ij}]$

$$a_{ij} = \begin{cases} \text{\# of adj. with } v_i, v_j \\ 0 \text{ no adj. with } v_i, v_j \end{cases}$$

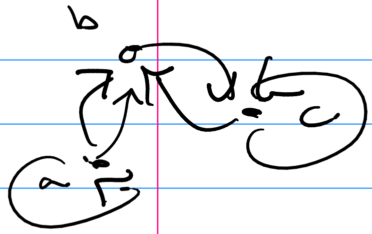
Ex $A_G = \begin{bmatrix} 0 & 0 & 2 \\ 0 & 1 & 1 \\ 2 & 1 & 0 \end{bmatrix}$

Note: $A_{\text{und. graph}}$ are symmetric

Directed Graph

(1) Adj. to/from List

initials	terminal



ex

	term
a	a, b
b	c
c	b, c

(2) Adj matrix. $A_G = [a_{ij}]$ $a_{ij} = \begin{cases} \# \text{ of adj. to} & (v_i, v_j) \in G \\ 0 & (v_i, v_j) \notin G \end{cases}$

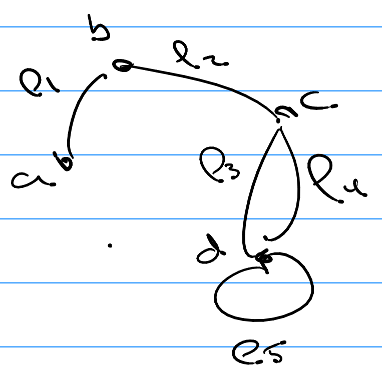
$$A_G = \begin{bmatrix} 1 & 2 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

Incidence Matrix:

$$I_G = \begin{array}{c} \\ v_1 \\ v_2 \\ \vdots \\ v_n \end{array} \begin{array}{c} e_1 \; e_2 \cdots \; e_m \\ \begin{bmatrix} & & \\ & & \\ & & \\ & & \end{bmatrix} \end{array}$$

$i_{ij} = \begin{cases} 1 & \text{if } v_i, e_j \text{ inc.} \\ 0 & \text{otherwise} \end{cases}$

ex



$$I_G = \begin{array}{c|ccccc} & e_1 & e_2 & e_3 & e_4 & e_5 \\ \hline a & 1 & 0 & 0 & 0 & 0 \\ b & 1 & 1 & 0 & 0 & 0 \\ c & 0 & 1 & 1 & 1 & 0 \\ d & 0 & 0 & 1 & 1 & 1 \end{array}$$

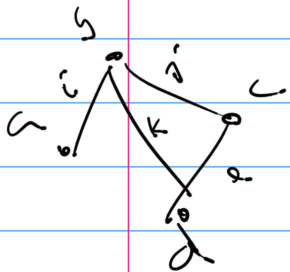
Note:

$$\underset{V \times E}{\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 \end{bmatrix}} \underset{E \times V}{\begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}} = \begin{array}{c} \\ a \\ b \\ \\ \end{array} \begin{array}{c} a \;\; b \;\; c \;\; d \\ \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ & & & \\ & & & \end{bmatrix} \end{array}$$

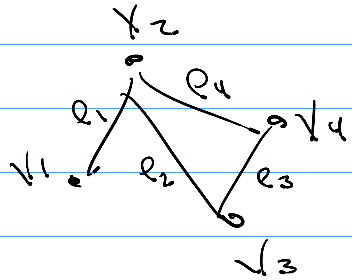
"Same" graphs

$G_1 = (V_1, E_1)$

$G_2 = (V_2, E_2)$



Same $\equiv \{V_1 = V_2\}$
$\{E_1 = E_2\}$



but ① if we pick any label?

② what if the vertices move graphically?

Isomorphic Graph

Def: G_1 is isomorphic to G_2 if a bijection from V_1 to V_2 exists and it preserves edges.

bijection: isomorphism

Show G_1, G_2 isomorphic

$$A_{G_1} = \begin{array}{c|c} & v_1 \; v_2 \cdots v_n \\ \hline \begin{matrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{matrix} & \begin{bmatrix} & & \\ & & \\ & & \end{bmatrix} \end{array}$$

Isomorphism

$f(v_1) =$

$f(v_2) =$

$\vdots$

$f(v_n) =$

$$A_{G_2} = \begin{array}{c|c} & f(v_1) \cdots f(v_n) \\ \hline \begin{matrix} f(v_1) \\ f(v_2) \\ \vdots \\ f(v_n) \end{matrix} & \begin{bmatrix} & & \\ & & \\ & & \end{bmatrix} \end{array}$$

if $A_{G_1} = A_{G_2}$

$\boxed{\text{Isomorphic}}$

How do find isomorphism or show not isomorphic?

→ Check Invariants under bijection.

① $|V_1| = |V_2|$ (no? → not isomorphic)
② $|E_1| = |E_2|$ (no? → not isomorphic)
③ $\deg(v)$ must be preserved (no? → not isomorphic)

④ deg of neighborhoods are preserved (no? → not isomorphic)

⑤ paths must be preserved (no? → not isomorphic)

Def: A path of length n is a seq of n edges in G.

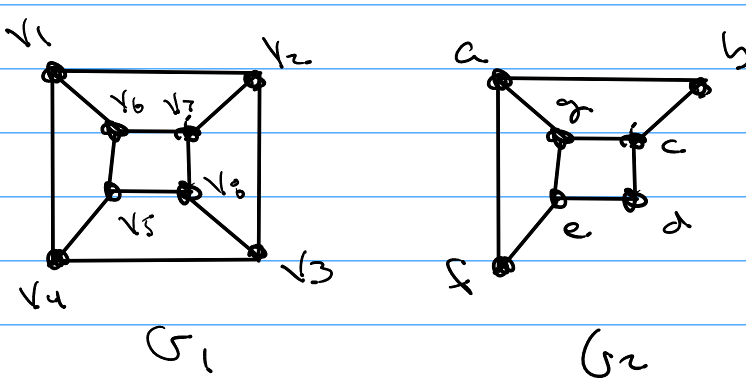
$$(x_0, x_1), (x_1, x_2), \dots, (x_{n-1}, x_n)$$

or

$$\{x_0, x_1\}, \{x_1, x_2\}, \dots, \{x_{n-1}, x_n\}$$

① Simple path ≡ each edge is used once in the path
② Circuit ≡ $x_0 = x_n$

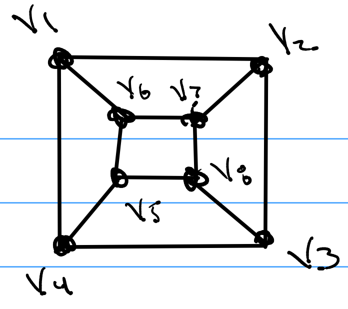
Ex



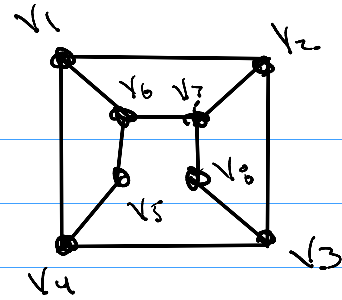
$|V_1| \neq |V_2|$

not isomorphic

ex



G_1



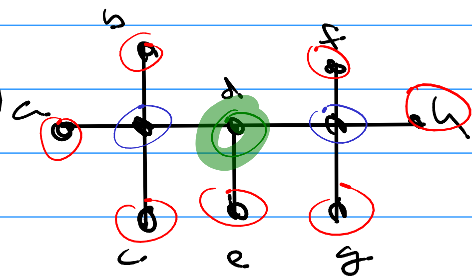
G_2

$|V_1| = |V_2|$

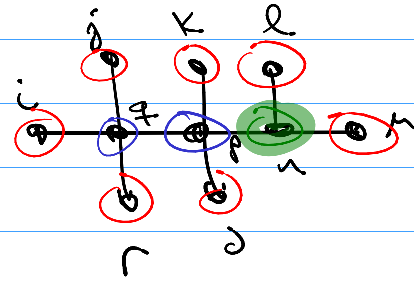
$|E_1| \neq |E_2|$

not isomorphic

ex



G_1



G_2

$|V_1| = |V_2|$

$|E_1| = |E_2|$

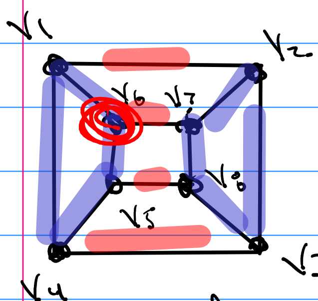
$\deg(n) = 3$

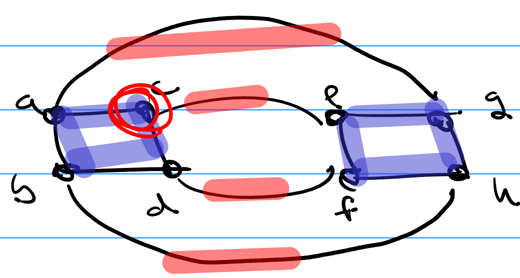
neighborhood → deg 1, 1, 4

$\deg(d) = 3$

neighborhood is deg 4, 4, 1

not isomorphic





	f	
v_1	→	a
v_2		
v_3		
v_4	→	b
v_5	→	d
v_6	→	c
v_7	→	e
v_8		

finish