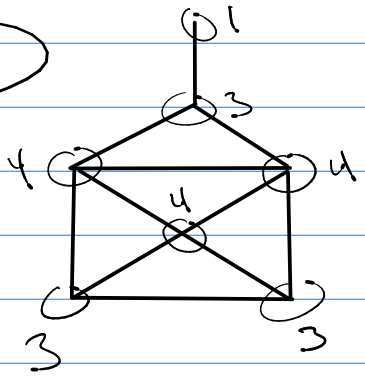
Math 322

Q5 (6.5 #15



#21 back to #16 #17

Thm G a directed multigraph

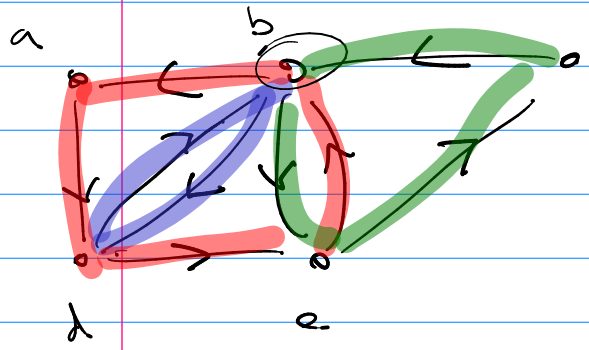
① Euler circuit (iff) weakly connected $\forall v \in V \quad \deg^-(v) = \deg^+(v)$

② Euler path (not circuit) (iff) weakly connected

for all (except two) $x \in V \quad \deg^-(x) = \deg^+(x)$

and on exactly two x_1, x_2

$$\deg^+(x_1) = \deg^+(x_2) + 1$$
$$\deg^-(x_2) = \deg^-(x_1) + 1$$



	In	Out
a	1	1
b	3	3
c	1	1
d	2	2
e	2	2

→ Euler Circuit.

d, b, d

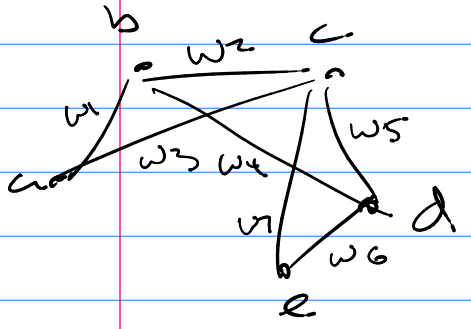
d, e, b, a, d

d, b, d, e, b, a, d

b, e, c, b

Shortest Paths

$G = (V, E)$ with $W : E \to \mathbb{R}$ ← weights



$w(e)$ = real number (positive)

Path: seq of edges

length = sum of weights for path.

Matrix $W_G = [w_{ij}]$

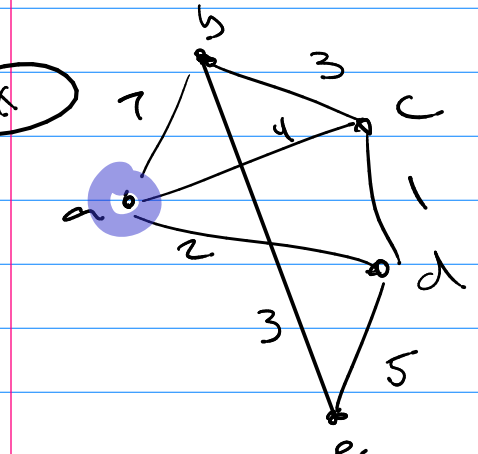
w_{ij} = weight of edges $\{v_i, v_j\}$ or (v_i, v_j)

$= \infty$ if $\{v_i, v_j\} \notin E$

---

(1) Find all shortest paths from some specific start vertex.

Ex



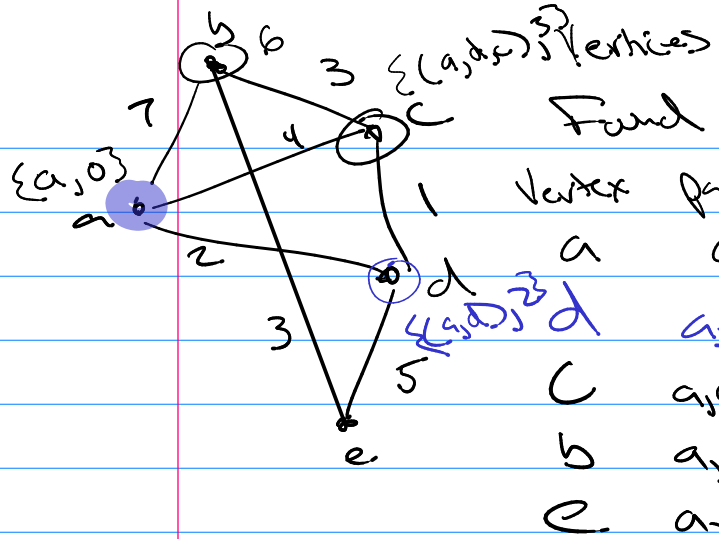
start @ a . . .

Shortest:

a to a
a to b
a to c
a to d
a to e

Sort

path to take and its cost.



Vertices Found

Vertex	path	cost
a	a	0
d	a,d	2
c	a,d,c	3
b	a,d,c,b	6
e	a,d,e	7

Vertices Not Found

∅

Dijkstra's

from set of **known vertices** list all possible next vertex to unknown vertex.

(+) costs, paths to them

→ find **least cost** - least cost (unique)

- not unique (add some extra reason to pick one)

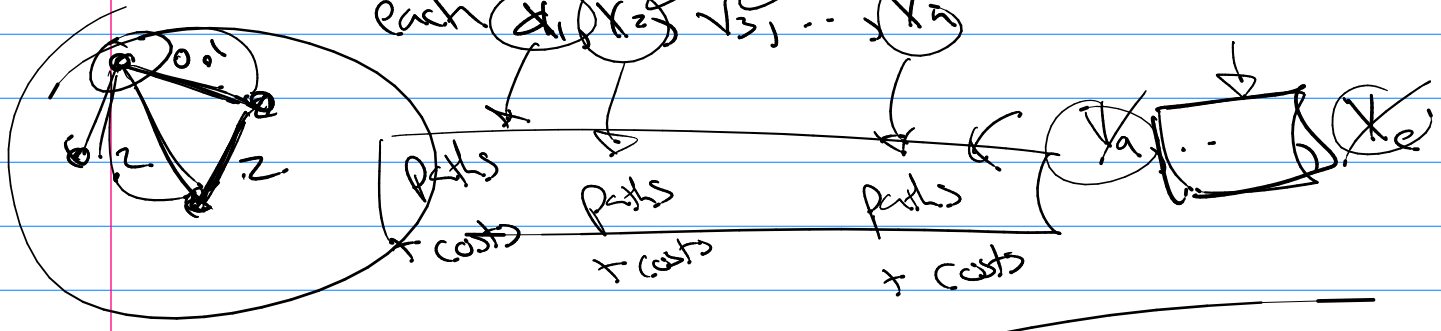
→ add to known

loop

Applications for dijkstra's

① find least cost paths starting @ v.

② Collect all the least cost paths for each $v_1, v_2, v_3, \ldots, v_3$

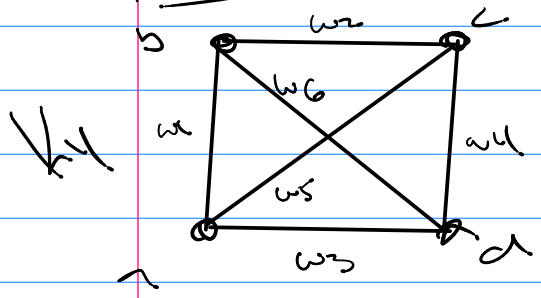


Traveling Salesman

↳ for a K_n graph we have $(n-1)!_6$ Hamilton circuits

→ $(n-1)!_6$ costs assoc. with the Hamilton circuits.

Q: least cost (shortest) Hamilton circuit.



K_4

Hamilton circuits $3!_6 = 6$

ok a, c, b, d, a | length = $w_5 + w_2 + w_6 + w_3$

a, d, b, c, a length =

for this b/c direction doesn't matter $\frac{(n-1)!_6}{2}$ uniq. paths.

→ must make all $\frac{(n-1)!_6}{2}$ circuits and their costs.

Min = 945

Soln: K_4 → paths $\frac{3!_6}{2} = 3$

K_5 → paths $\frac{4!_6}{2} = 12$

K_6 → paths $\frac{5!_6}{2} = 60$

Exam — 12 probs @ 10 pts each
110 pts = 100%

10.1 Intro (2 probs)

① (multi part) Name / Draw graph types & properties (|V|, |E|, degrees)

② Know: Influence Graphs, Niche Overlap Graphs, and Intersection Graphs

10.2 terms, theorems, and special graphs (2 probs)

① ② Know K_n, C_n, W_n, Q_n, $K_{n,m}$
Coloring th^m, Handshake th^m

10.3 Isomorphism (2 probs)

① ② Isomorphic?

10.4 Connectivity (2 probs)

① $\kappa(G)$, $\tau(G)$

② Directed Graph: Q on connectedness

10.5 Euler / Hamilton

①② Euler probs

③ Hamilton (know ORE's, Dirac's)

16.6 Shortest Paths (1 pair)

① Dijkstra (use it)