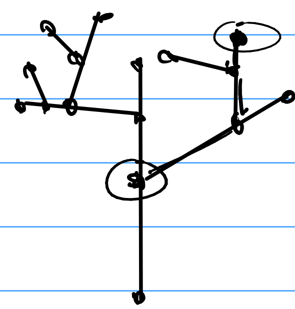
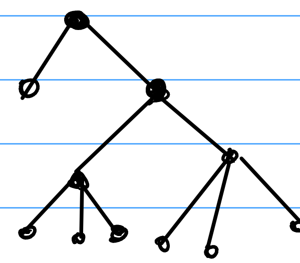
Math 322

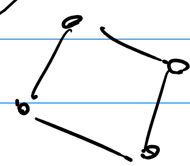
$G = (V, E)$

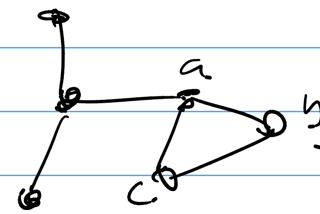
Ch 11 trees





not trees





a, b, c, a ← simple circuit

Def: tree is a connected undirected graph with no simple circuits

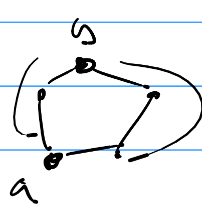
thm G is a tree **iff** there is a unique simple path between any two of its vertices.

→ Same as .. (let's assume connected undirected graphs)

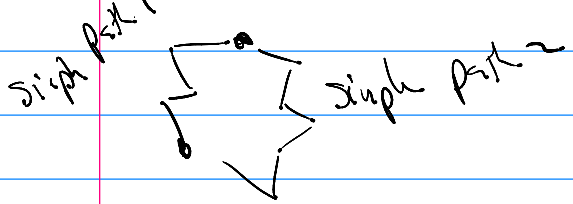
"no simple circuits **iff** unique simple path between any two vertices"

→ Same as .. "simple circuit iff ≥ 2 or more simple paths between two vertices"

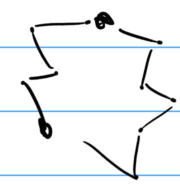
pf case "left → right"

assume simple circuit  show 2 or more simple paths (take left or right)

"right → left"



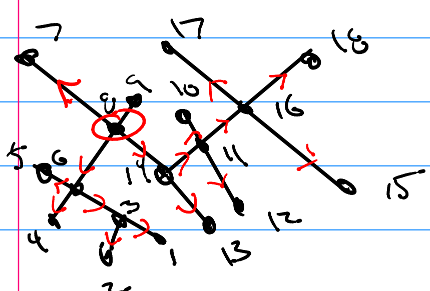
→ if (is) a simple circuit



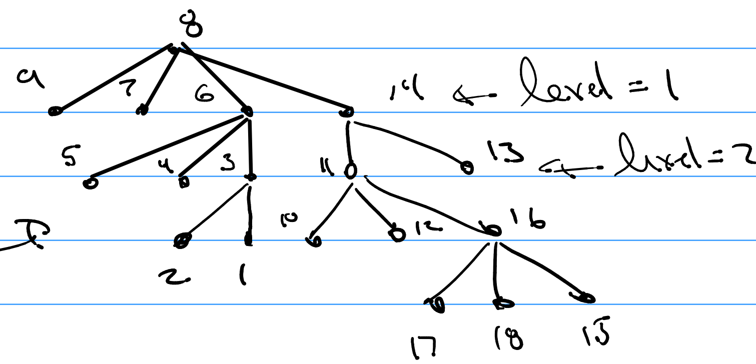
Types of trees

undirected

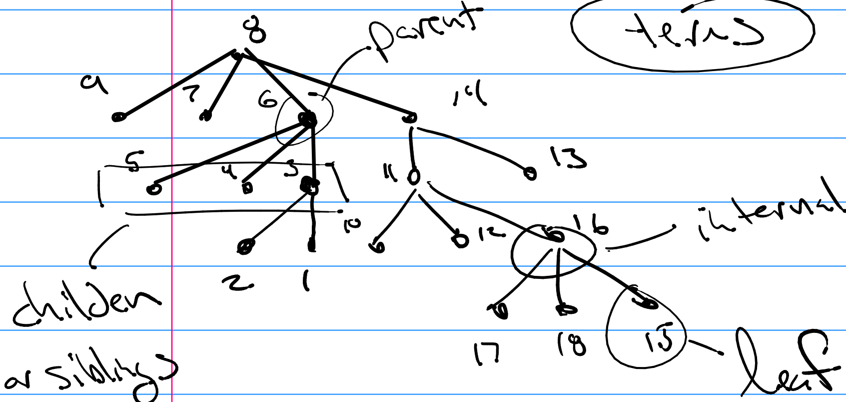
Def A rooted tree is a tree where you call a vertex a root and direct every edge away from the root.



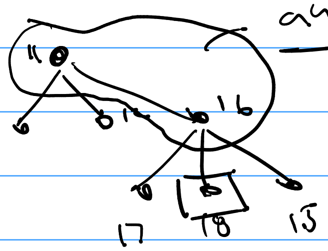
h=4

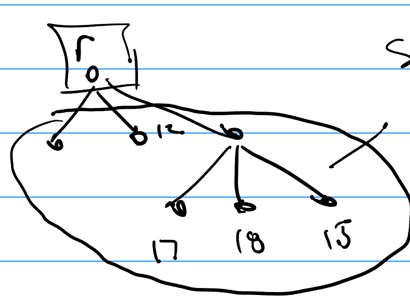


terms



ancestors (root to your parent)





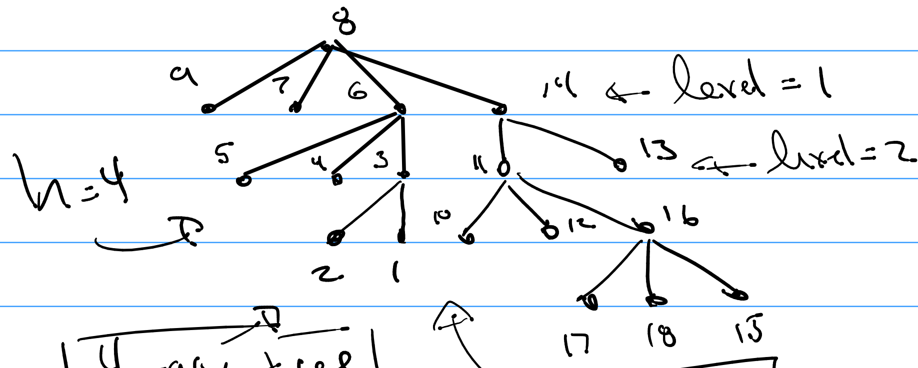
set of all vertices w/ r as an ancester : descendants

$\boxed{\text{more terms}}$ ① level = length of uniq. path to root.

② height = h = max level

③ $\boxed{\text{M-ary}}$ tree

m = largest group of childern



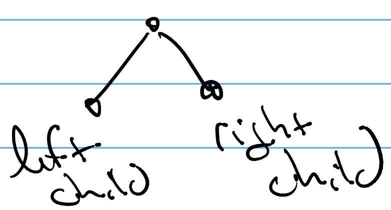
$\boxed{\text{4-ary tree}}$

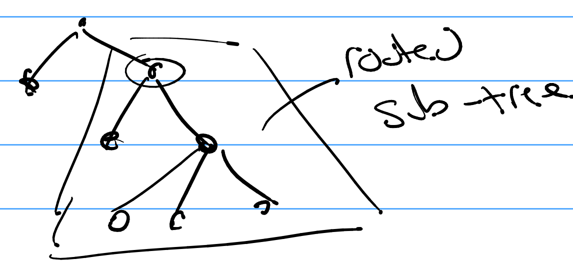
$\boxed{\text{not full}}$

④ $\boxed{\text{full M-ary}}$ tree

all parents have m children

$\boxed{\text{Def}}$ ordered rooted tree. Rooted tree, but children are ordered.





Def: rooted sub-trees

sub-graph taking a vertex as a root and all its descendents.

Properties of trees (thm's) $T=(V,E)$

① $n=|V|$ $\qquad n = \underset{\text{internal}}{i} + \underset{\text{leaves}}{l}$

② Thm: $T=(V,E)$ is a tree with $|V|=n$

a) $|\text{children}| = n - 1$ ← root

b) $|E| = n-1$

③ Thm: T is a full m-ary tree with i internal vertices

a) $|\text{children}| = i \cdot m$

b) $n = mi + 1$

Thm #4 (full m-ary tree) Know: $\begin{cases} n = i + l \\ n = mi+1 \end{cases}$ n, i, l are unknown

(i) given n → solve for i, l

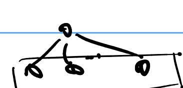
(ii) given i → solve for n, l

(iii) given l → solve for n, i

⑤ Thm: T is of height h, there is at most m^h leaves

$$\boxed{l \le m^h \qquad h = 0, 1, 2, \ldots}$$

Pf) Basis: $(h=0) \to T$ is $\bullet$?? $1 \le m^0$ true!

look $(h=1) \to T$ is  ?? $l \le m^1$ true

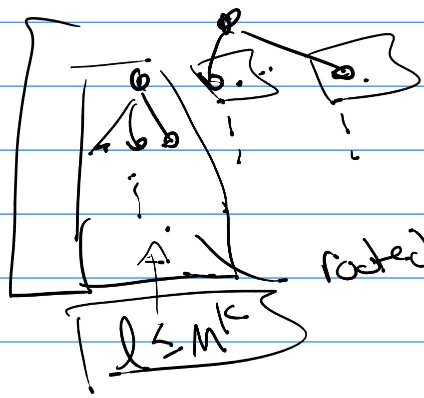
children $\le m$

Inductive

assume: $h=k$ $\quad l \le m^k$ (tree of height k)

show for tree of $h=k+1$

$h=k+1$



rooted subtrees of 1^{st} children are $h=k$

at most $m \cdot m^k$ leaves $\quad m^{k+1}$

Def balanced tree has all leaves at level h or $h-1$.

So

$$l \le m^h$$

$$\log_m l \le h$$

Corollary

so $\quad h \ge \lceil \log_m l \rceil$

full and balanced $\quad h = \lceil \log_m l \rceil$

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text. $\quad$ fwd $\;h=5$ $\quad$ fwd $\;h=4$ $\quad$ fwd $\;h=3$ $\quad$ fwd $\;h=2$

blah blah

send to 10 people b/c!

$.25

sender

edge = $.50

rec.

full 10-ary tree

how much money $= \boxed{|E|}(\$.50)$

$|E| = n - 1$

↑ number of people in tree

assume: full 10-ary tree & $h = 5$ money $= 50¢(n-1)$?

$\boxed{l \le 10^5}$

worst case assumption $l = 100{,}000$ (get, not send)

$$\begin{cases} n = i + 10^5 \\ n = 10i + 1 \end{cases}$$

$9i = 10^5 - 1$

$i = \frac{10^5 - 1}{9} = \frac{99{,}999}{9} = \boxed{11{,}111}$ senders.

$n = 111{,}111$ $\$ = 50¢(111{,}110)$
