

Math 322

Q2

$n = |V|$

$n = |L| + |R|$
 (L: kids, R: have no kids)
 $n = |M| + 1$
 $n - 1 = |E|$

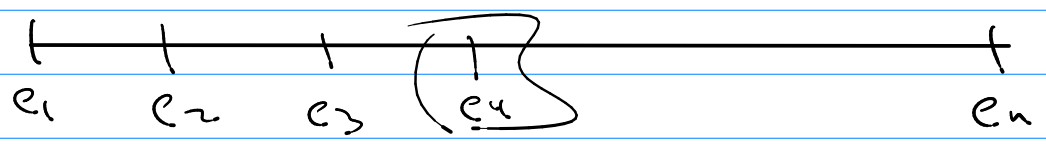
full

11.2

Applications of Trees

1 Binary Search Trees

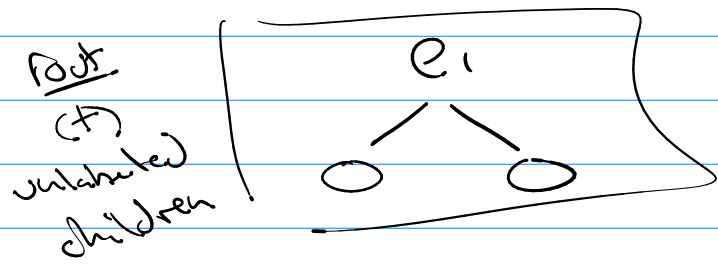
2 total ordering \rightarrow well ordered



cost = $n - \underline{\underline{compare}}$

Binary Search tree

Set = $\{e_1, e_2, e_3, \dots, e_n\}$

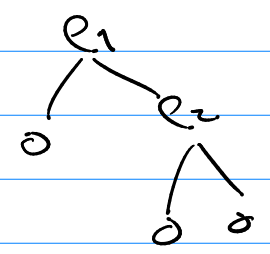
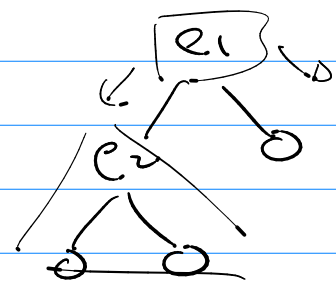


compare $e_2 < e_1$

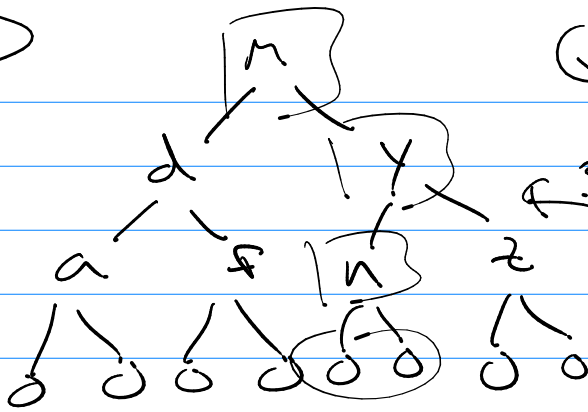
$e_2 < e_1$

$e_2 > e_1$

loop



ex



(15)

$\{a, d, f, n, n, y, z\}$

3 compares

7 compare

Now add 1 element

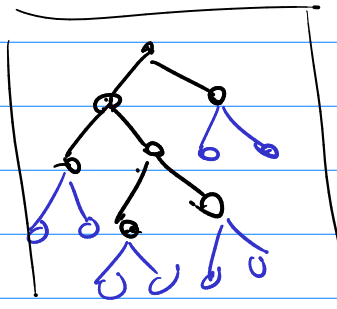
K elements

Cost for linear: K compares

Cost for a binary search tree?

→ K elements
→ add unlabeled vert.

$i = K$



$n=2$

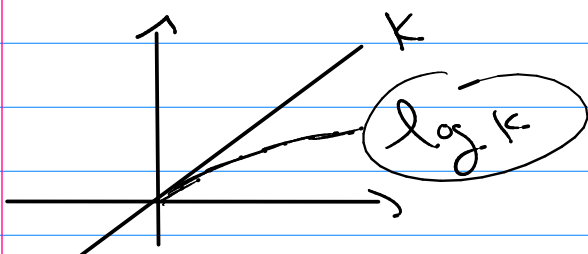
$$\begin{cases} n = K+1 \\ n = 2K+1 \end{cases} \rightarrow \boxed{l = K+1}$$

$$h \geq \lceil \log_2(K+1) \rceil$$

balanced $h = \lceil \log_2(K+1) \rceil$

put stuff in linear order: cost K compares

put stuff in binary search tree: cost $\lceil \log K \rceil$

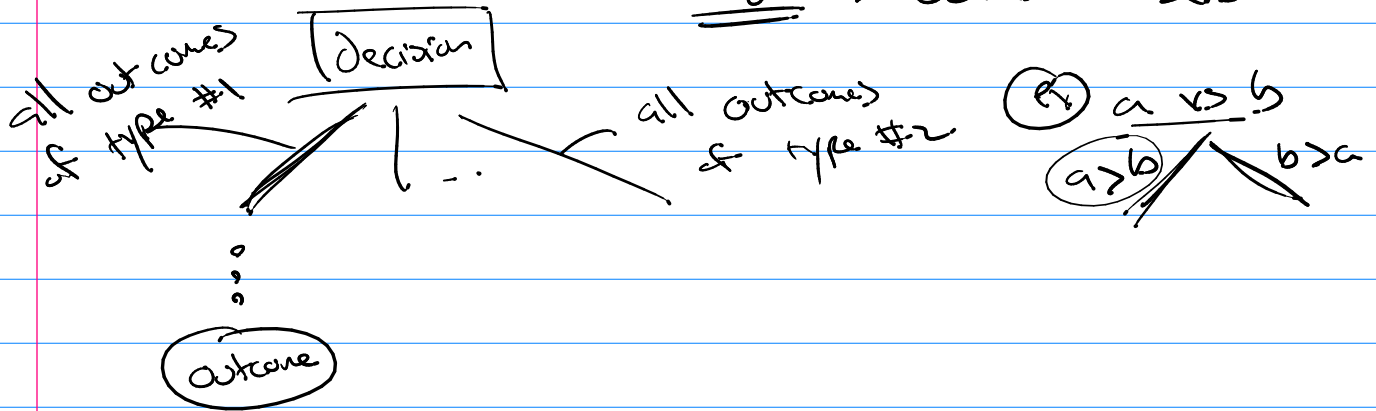


Decision Trees

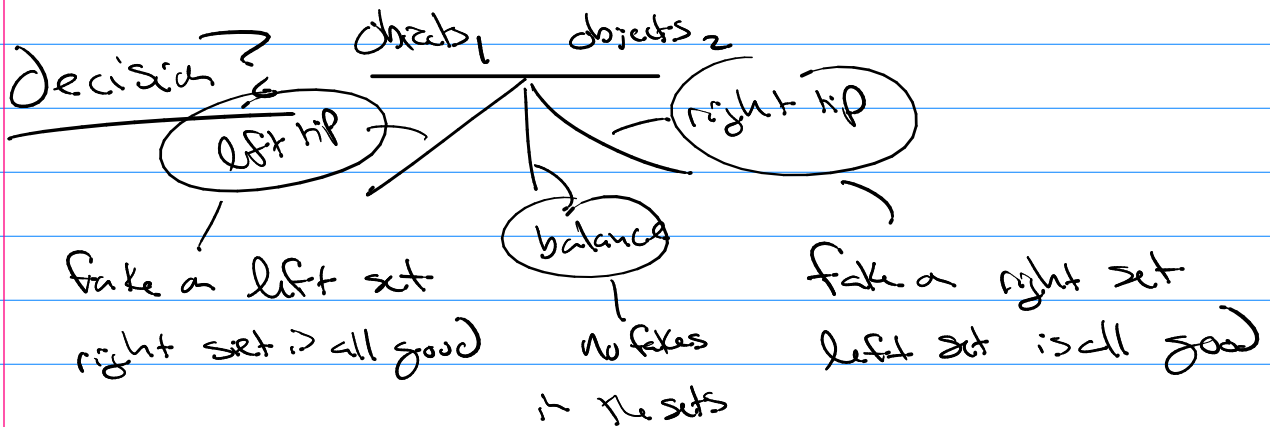
(e) : outcome

[i] : decision process

edges : outcome sets



Test balance scale ; 4 coins ; 1 may be a heavy fake.



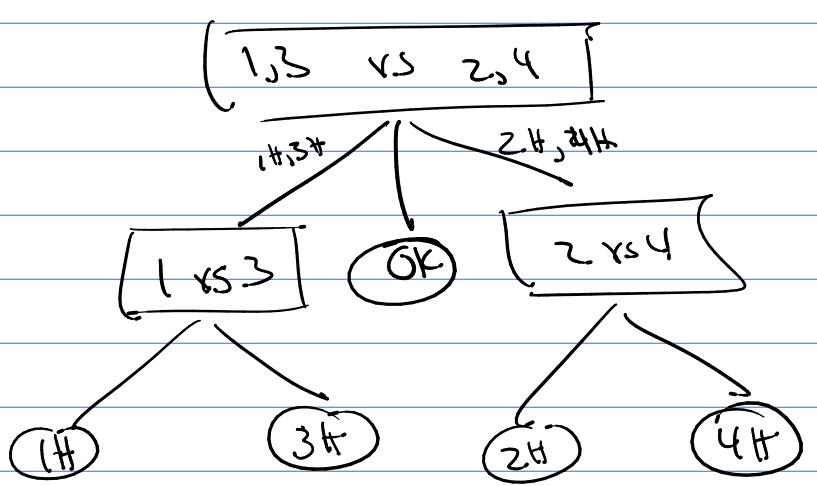
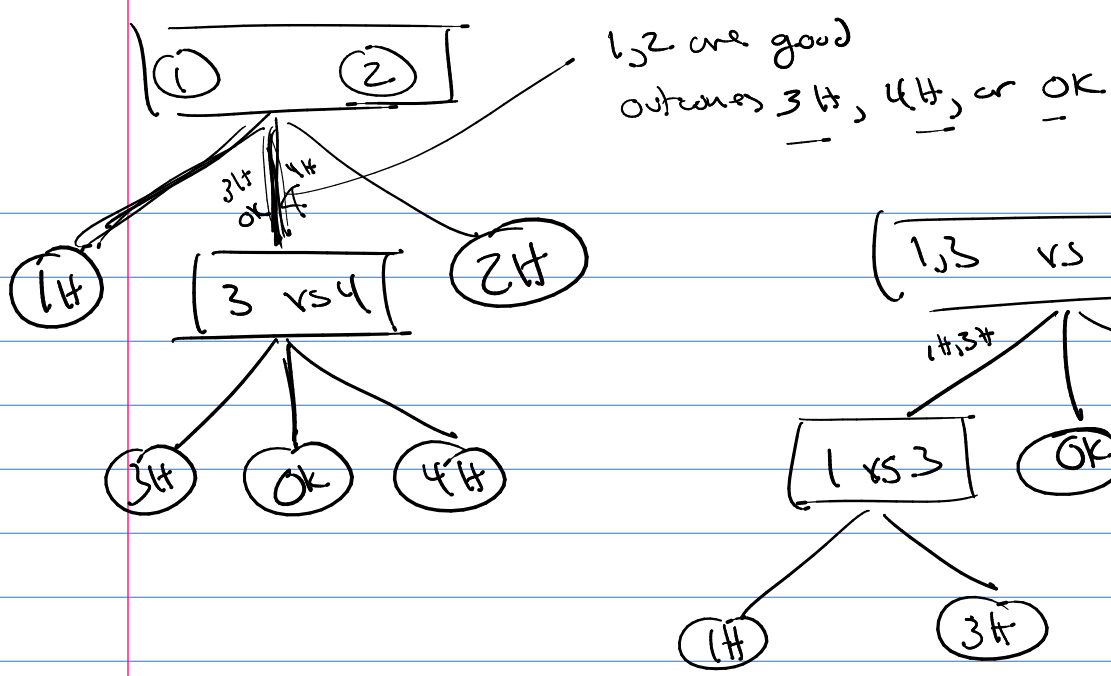
Outcomes? (1H), (2H), (3H), (4H), (OK)

$$\left. \begin{array}{l} l = 5 \\ n = 3 \end{array} \right\} h \geq \lceil \log_3 5 \rceil = 2$$

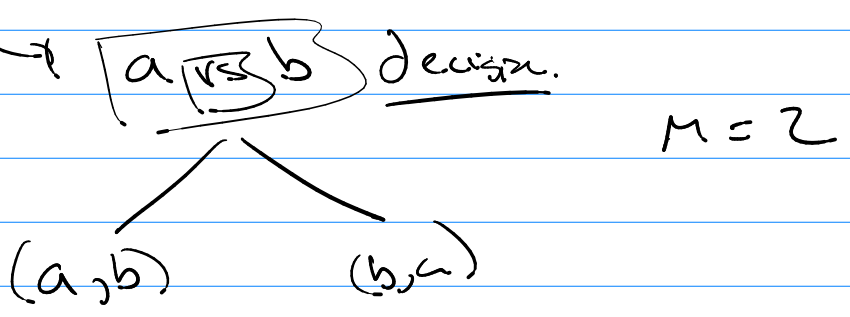
Make tree (1), (2), (3), (4)

1 vs 1 ?

2 vs 2 ?

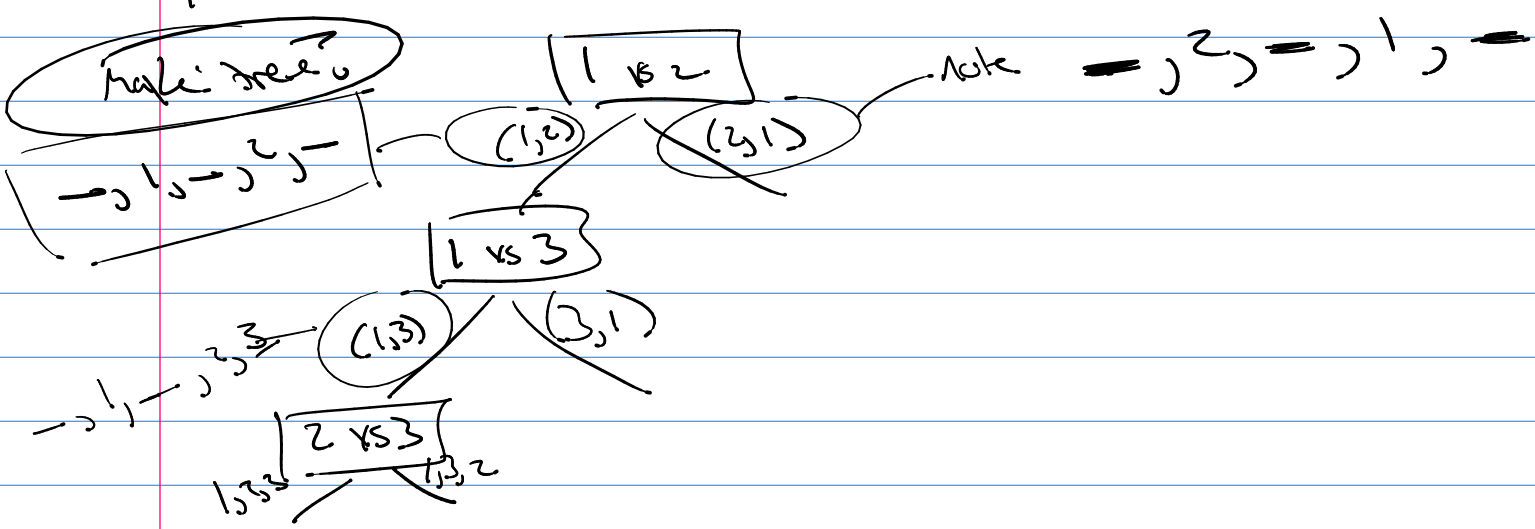


Linear Sort of objects $O_1, O_2, O_3, O_4 \rightarrow (1, 2, 3, 4)$ possible sort



outcomes ? $4 \cdot 3 \cdot 2 \cdot 1 = 4! = 24 = 2^5 = 2$

So $h \geq \lceil \log_2 24 \rceil = 5$



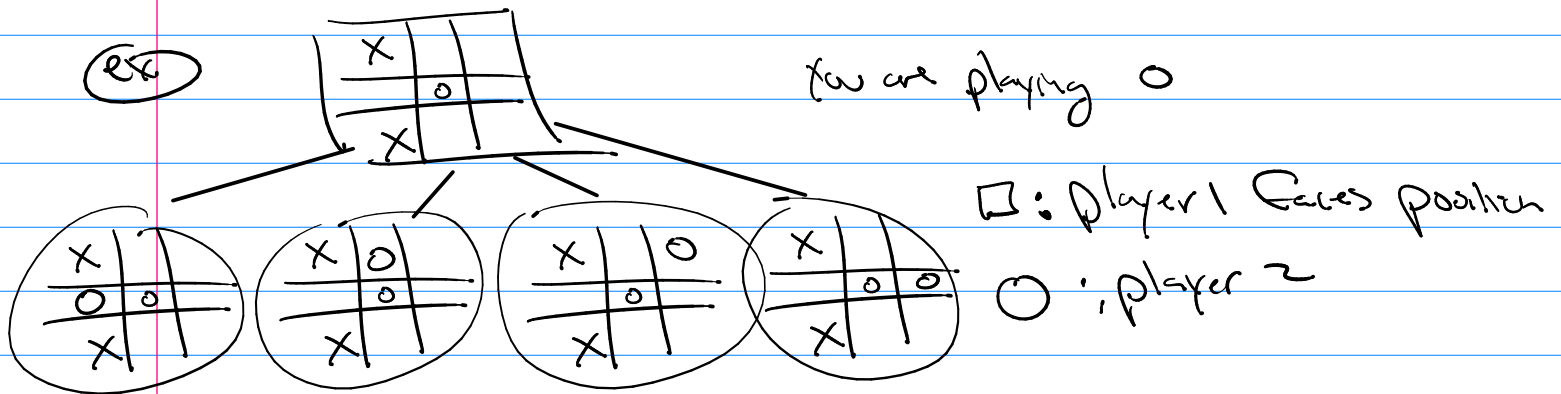
Game trees

two players, take turns

Decision (internal vertex) = game position (non-end)

leaf = end game position

edge = next move (next allowed positions)

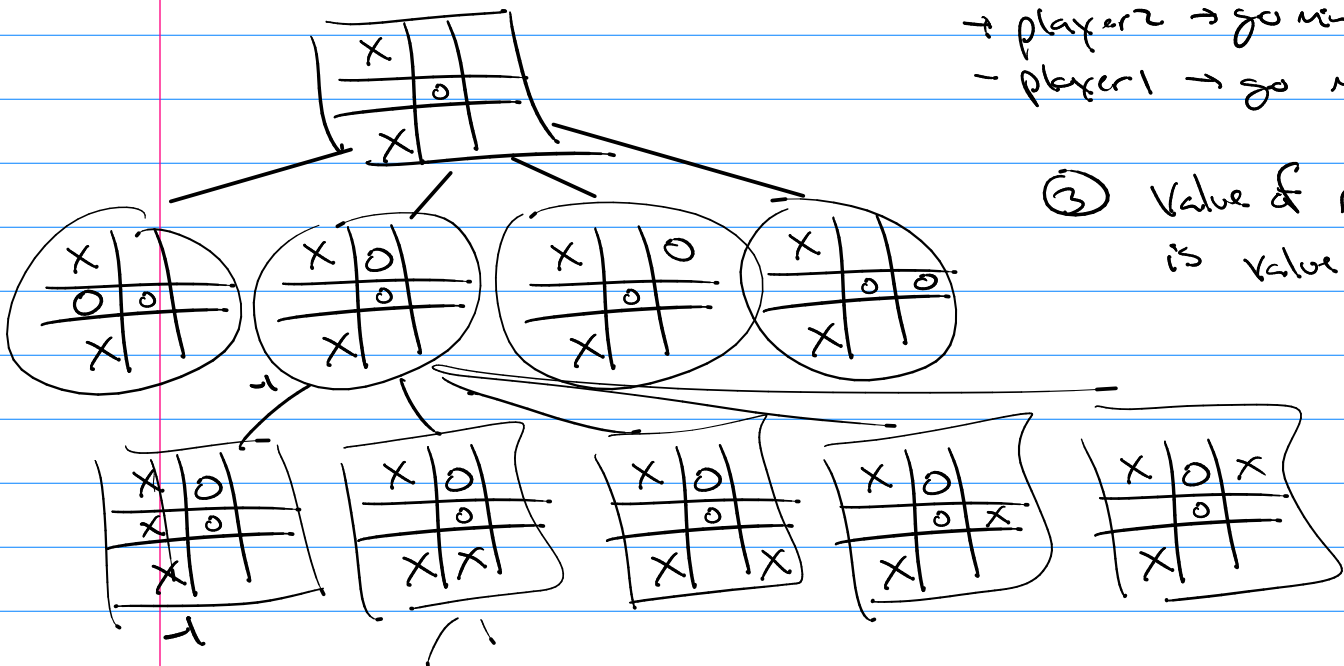


→ label the vertices? ① leaf = put the return to player 1

② internal vertex: (min max strategy)

- player 2 → go min child
- player 1 → go max child

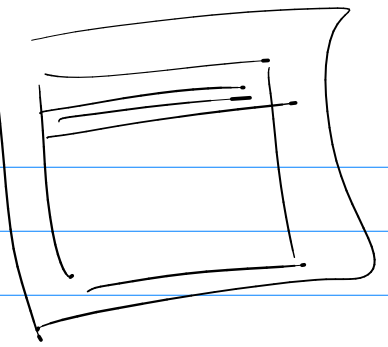
③ value of root is value of game



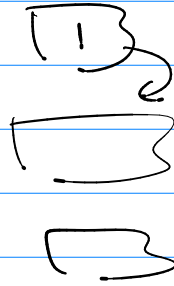
Complex games

Chess:

8x8



20-ary.

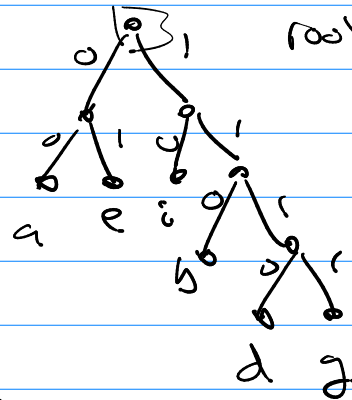


$l=1 \quad |c|=20$

$l=2 \quad |c|=20^2$

Prefix Codes

- label leaves
- label edges
- left = 0
- right = 1



root to a

uniq simple path = 00

root to d = 1110

dad = 1110001110

1101001110 = bad