

# Math 322

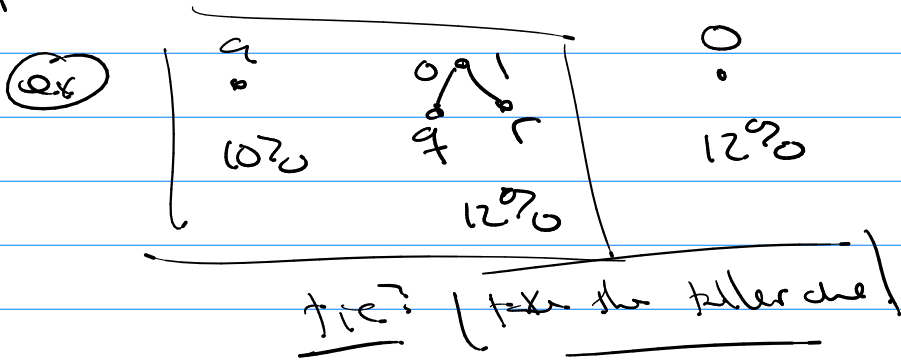
Labels are leaves only.

Prefix Code

Huffman Code

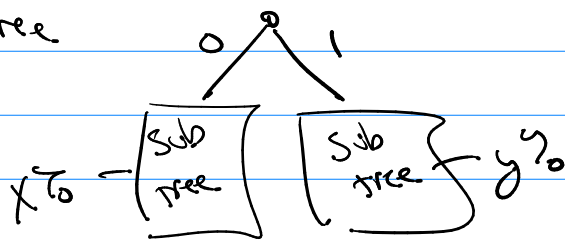
- (1) subtree 1, subtree 2, subtree 3  
" " "  
 $x\%$   $y\%$   $z\%$

(2) pick the two  $\%$ 's that are least

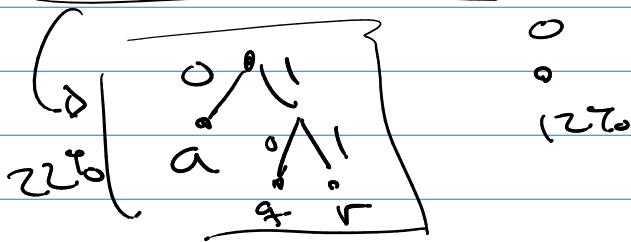
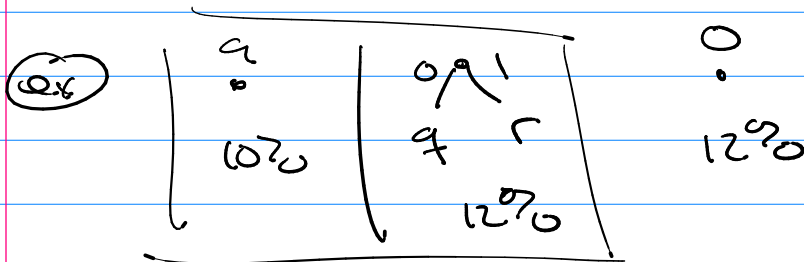


(3) make a new subtree

if  $x < y$



new  $\%$  of subtree =  $(x+y)\%$



(4) a, e, i, o, d, b, f  
 20% 18% 12% 25% 10% 8% 7%

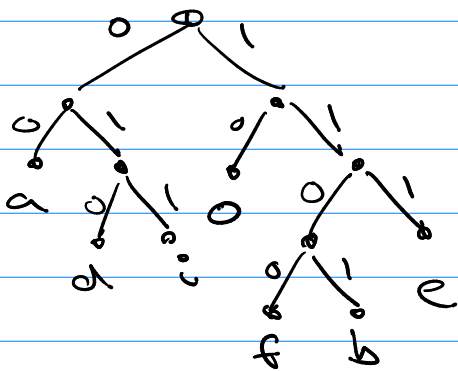
(1) a, e, i, o, d, f b 15%  
 20% 18% 12% 25% 10%

(2) a, e, o, d i 22%, f b 15%  
 20% 18% 25%

(3) a o 20% 25%, d i 22%, f b e 33%  
 20% 25%

(4) o 25%, f b e 33%, a d i 42%

(5) (6)  
 f



prefix code / Huffman code

a: 00      d: 010  
 o: 10      f: 1100  
 e: 111     b: 1101  
 i: 011

dad: 01000010

Game tree

(ex)

Variant of Nim.

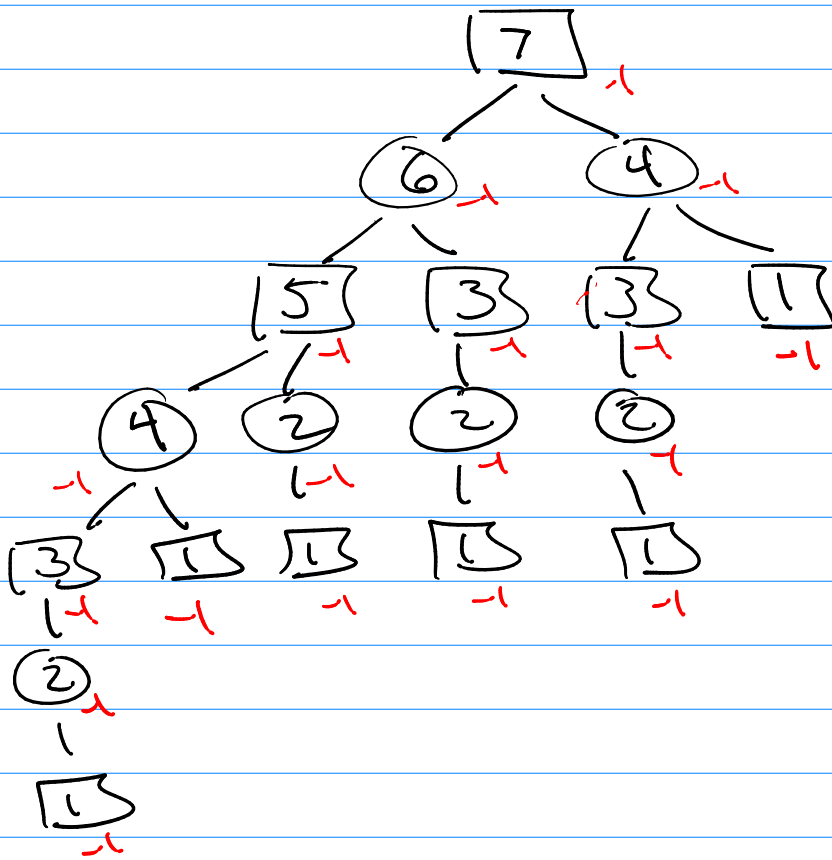
(take last object (or all) = lose)

Regr: a) 1 pile

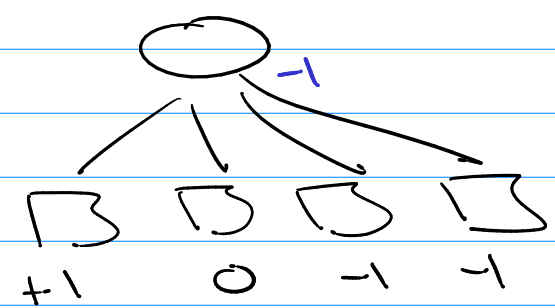
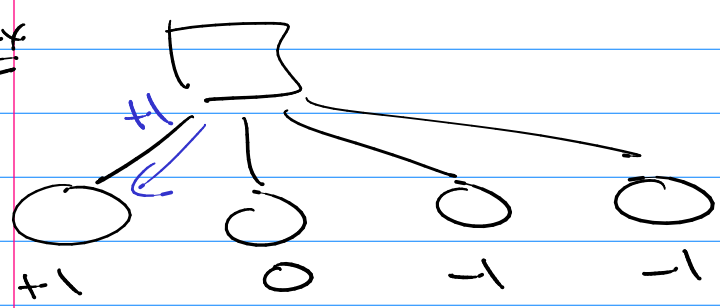
b) take 1 or 3

(ex)

Start with 7 stones

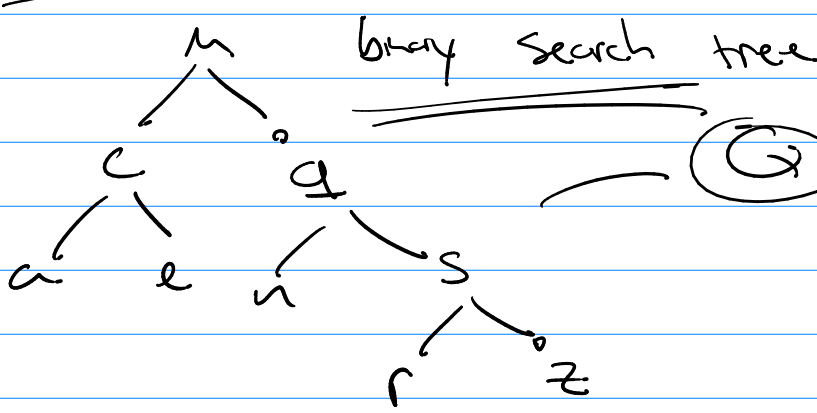


Minimax

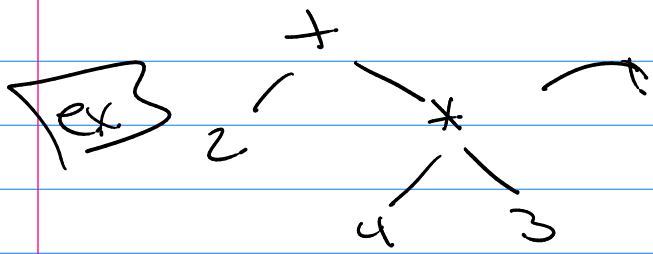


# getting elements out of trees

(ex)



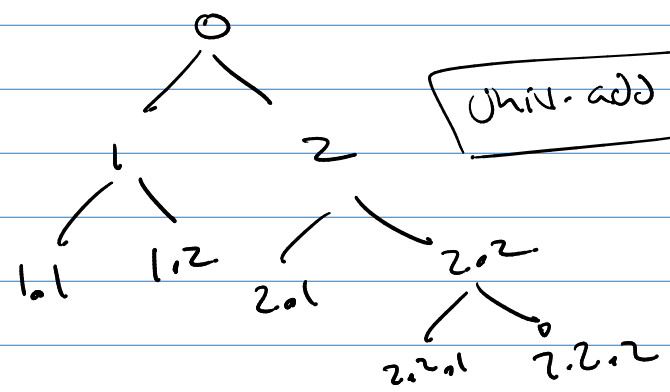
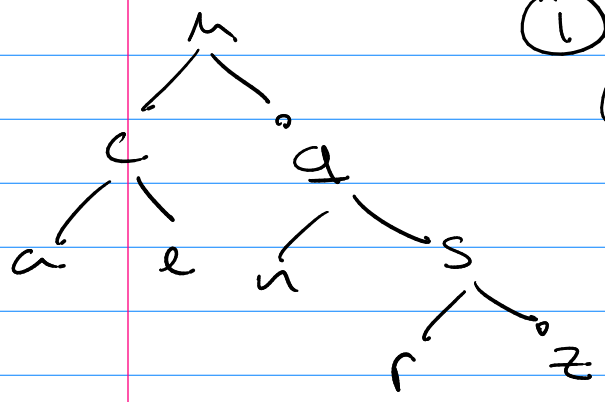
linear form?



linear list of nodes

## tree traversals

① Universal Address System (plus) lexicographic ordering

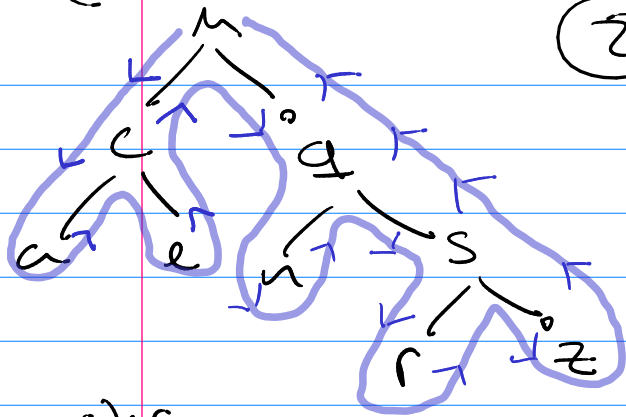


Univ. add.

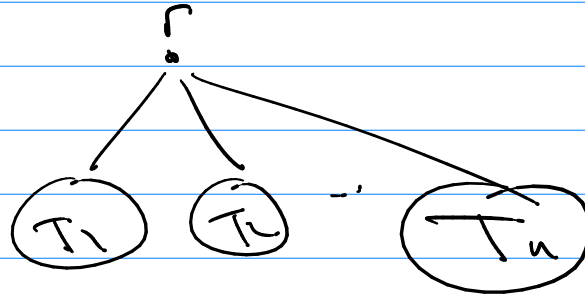
lexicographic sort

- 0, 1, 1.1, 1.2, 2, 2.1, 2.2, 2.2.1, 2.2.2
- [m, c, a, e, g, n, s, r, z]

(traversals)



## ② Pre-Order Traversal

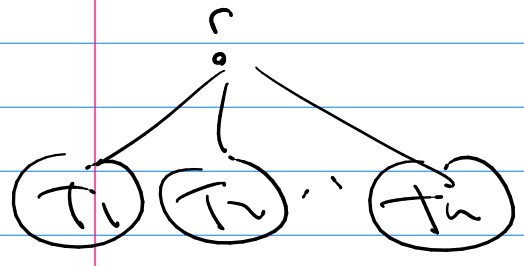


pre-order

m, c, a, e, q, n, s, r, z

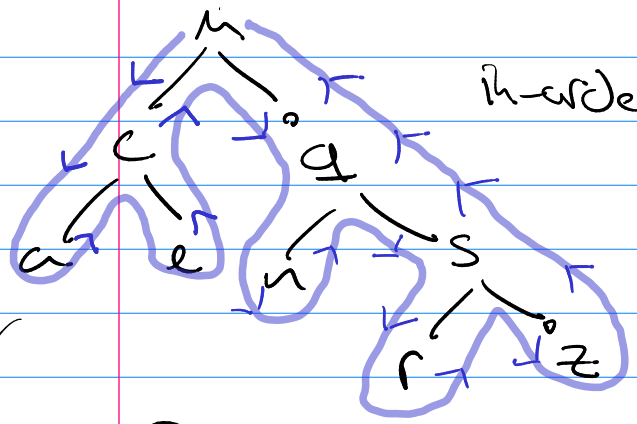
- ① Visit r
- ② Visit T<sub>1</sub> in pre-order
- ③ Visit T<sub>2</sub> in pre-order
- ⋮
- ④ Visit T<sub>n</sub> in pre-order

## ③ In-Order Traversal



- ① Visit T<sub>1</sub> in in-order
- ② Visit r
- ③ Visit T<sub>2</sub> in in-order
- ⋮
- ④ Visit T<sub>n</sub> in in-order

in-order: a, c, e, m, n, q, r, s, z



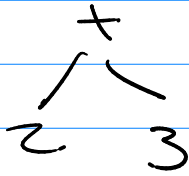
## ④ Post-Order Traversal

post order: a, e, c, n, r, z, s, q, m

- ① Visit T<sub>1</sub> in post-order
- ② Visit T<sub>2</sub> in post-order
- ⋮
- ④ Visit T<sub>n</sub> in post-order
- ⑤ Visit r

# Applications of traversal : Math operations

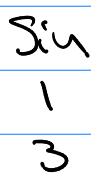
(a) Binary operator



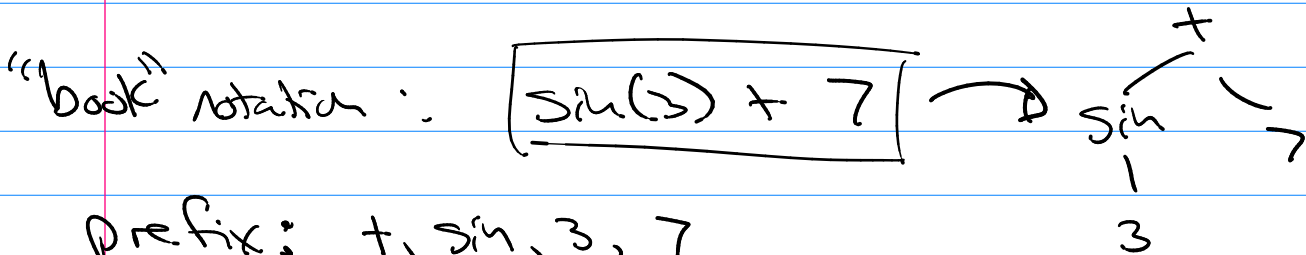
Linear rep. of math operations

pre-order :	$+ , 2 , 3$	← prefix notation
in-order :	$2 , + , 3$	← infix notation
post-order :	$2 , 3 , +$	← postfix notation

(b) Unary operator:



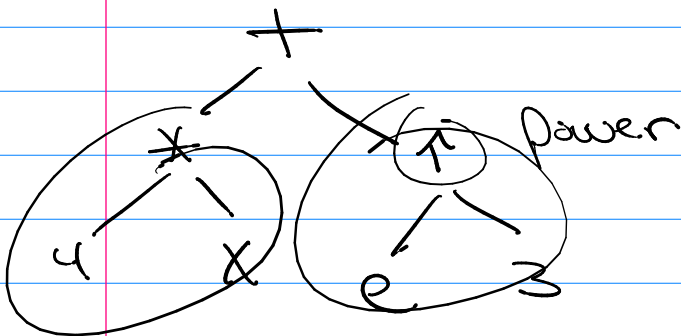
Prefix :  $\sin , 3$   
 infix :  $3 , \sin$   
 postfix :  $3 , \sin$



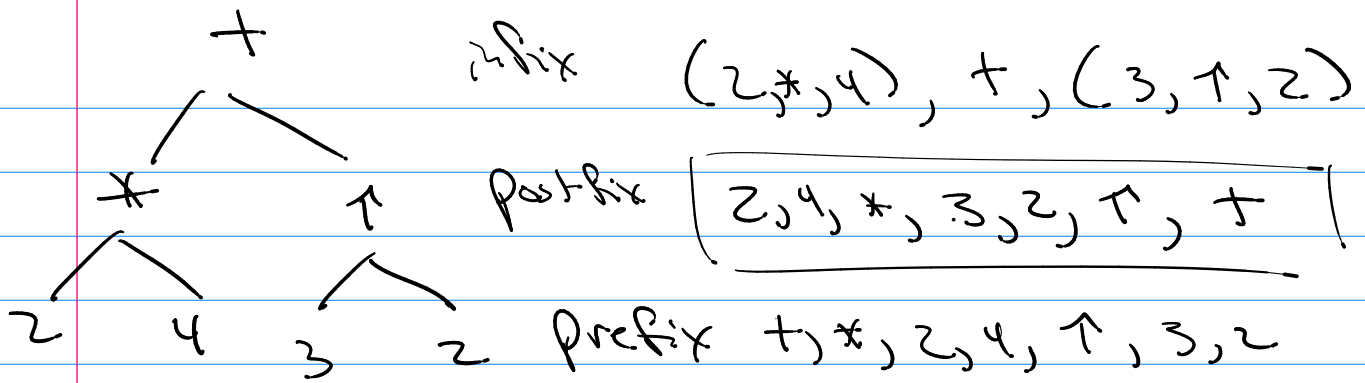
Prefix :  $+ , \sin , 3 , 7$

Infix :  $3 , \sin , + , 7$

Postfix :  $3 , \sin , 7 , +$



Prefix :  $+ , * , 4 , x , ^ , e , 3$   
 Postfix :  $4 , x , * , e , 3 , ^ , +$   
 infix :  $(4 , * , x) + (e , ^ , 3)$   
 ↑  
 must have grouping around subtrees



Jan Łukasiewicz

prefix : Polish notation

postfix : reverse polish notation

