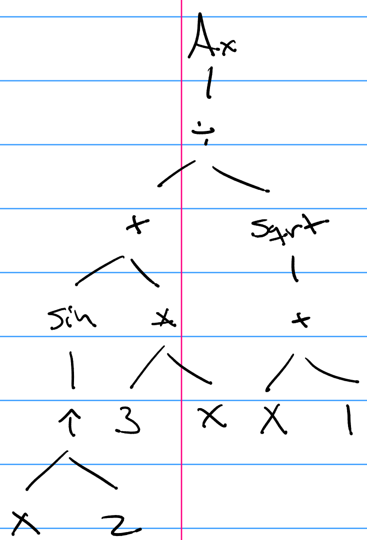
Math 322

Q's $A_x[f(x)] = \int f(x)\,dx$

$$A_x\left[\frac{\sin(x^2)+3\cdot x}{\sqrt{x+1}}\right]$$

pre-fix, post-fix, in-fix



postfix: x, 2, ↑, sin, 3, x, *, +, x, 1, +, sqrt, ÷, A_x

prefix: A_x, ÷, +, sin, ↑, x, 2, *, 3, x, sqrt, +, x, 1

Infix: ((((x, ↑, 2), sin), +, (3, *, x)), ÷, ((x, +, 1), sqrt)) A_x

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Ch 12 Boolean Algebra

How low to go? Addition

1+1=2	$\boxed{2+2=4}$	axiom? or is there a "lower" axiom that makes (prove) this.
1+2=3	3+2=5	
1+3=4	4+2=6	

Axioms: 1 and count.   11+11 $\xrightarrow{\text{count}}$ 1111

1, 11, 111, 1111,

Boolean Algebra

objects : Set $B = \{e_0, e_1\}$

operations : ① two binary ops $\wedge$, $\vee$

② one unary op $\ominus$

Notation: $b \wedge e$, $b \vee e$, $\overset{\ominus}{b}$

the following laws hold:

① Identity Law

$$x \wedge e_1 = x$$

$$x \vee e_0 = x$$

② Complement Law

$$x \wedge \overset{\ominus}{x} = e_0$$

$$x \vee \overset{\ominus}{x} = e_1$$

③ commutative

④ associative

⑤ distributive:

$$x \vee (y \wedge z) = (x \vee y) \wedge (x \vee z)$$

$$x \wedge (y \vee z) = (x \wedge y) \vee (x \wedge z)$$

What are boolean Algebras?

① Prop. Logic $\quad B = \{T, F\}$

binary ops : and, or — inclusive or $\quad \wedge, \vee$

unary op : not $\quad \neg$

show $T \vee F \equiv T$

② Boolean Algebra (C.S. / circuits)

$B = \{0,1\}$ ← bits ops $+, \cdot, \overline{}$

x	y	$x+y$	$x \cdot y$	$\bar{x}$	$\bar{y}$
1	1	1	1	0	0
1	0	1	0	0	1
0	1	1	0	1	0
0	0	0	0	1	1

→ **Boolean Expressions:** (collection of variables, bits, ops)

ex $(x \cdot y) + ((z + \bar{0}) \cdot 1)$

ex $(x+y)\cdot z = (x \cdot z) + (y \cdot z)$

ex $(x \cdot y) + z = (x+z)\cdot(y+z)$

→ Boolean Function $f : B^n \to B$ ← how many vars?

ex $f(x,y,z) = (x \cdot y) + z$ $f : B^3 \to B$

→ tables

x	y	z	$x \cdot y$	$(x \cdot y) + z$
1	1	1	1	1
1	1	0	1	1
1	0	1	0	1
1	0	0	0	0
0	1	1	0	1
0	1	0	0	0
0	0	1	0	1
0	0	0	0	0

$(1,0,0) \xrightarrow{f} 0$

$(0,0,1) \xrightarrow{f} 1$

So $\boxed{\text{any}}$ Boolean function $f : B^{n} \to B = \{0,1\}$

n ← Variables

x_1	x_2	x_3	--	x_n	f
1	1	1		1	☐ 0 or 1
1					☐ 0 or 1
⋮					⋮
0	0	0	--	0	☐ 0 or 1

rows $= 2^n$

all possible tables

$$2^{(\#\text{ of rows})} = 2^{(2^n)}$$

$$|f : B^n \to B| = 2^{(2^n)}\ \boxed{\text{uniq}}\ \text{functions}$$

$\boxed{\text{But}}$ we have countable infinite expressions

(Next section: "best" expressions?)

Identities

1. Double complement $\overline{\overline{x}} = x$
2. Idempotent $x + x = x$, $x \cdot x = x$
3. (*) Identity $x + 0 = x$, $x \cdot 1 = x$
4. Domination $x + 1 = 1$, $x \cdot 0 = 0$
5. (*) Commutative
6. (*) Assoc
7. (*) Distrib
8. De Morgans
9. Absorption $x + (x \cdot y) = x$
   $x \cdot (x + y) = x$
10. (*) Complement / Negation / unit-zero laws
    $x + \overline{x} = 1$
    $x \cdot \overline{x} = 0$

show laws using only the ★ five?

ex $\boxed{X + X = X}$

$X = X + 0$	$X = X \cdot 1$	identity
$= X + (X \cdot \overline{X})$	$= X \cdot (X + \overline{X})$	complement
$= (X+X) \cdot (X + \overline{X})$	$= (X \cdot X) + (X \cdot \overline{X})$	distrib.
$= (X+X) \cdot 1$	$= (X \cdot X) + (0)$	complement
$= X + X$	$= X \cdot X$	identity

$X + X = X$ and $X \cdot X = X$