Math 322

Facebook: Date – stamp

\# ⋮

Stamp

people $\left[\ C\ \right]$

people

$C \cdot C^T \to$ people $\{\quad\}$

\# common stamps

p_1 — p_2

→ world of switches $B = \{0,1\}$

Computer $\boxed{f : B^n \to B}$

x_1	x_2	$\cdots$	x_n	f
1	1	$\cdots$	1	0 or 1
				⋮
0	0	$\cdots$	0	0 or 1

$|\text{Domain}| = 2^n.$

total # of unique functions $= \boxed{2^{(2^n)}}$
(finite)

Goal: f = expression of $+, \cdot, \bar{\ }, x_1, x_2, \ldots, x_n$

countable infinite.

Concepts

(1) $x_1 \cdot x_2 \cdot x_3 \cdot \ldots \cdot x_n$

a) 0 is the dominator (if any is a zero, entire product = 0)

b) Product is 1 if and only if all are 1.

(2) $x_1 + x_2 + \dots + x_n$

a) 1 is dominator (if any is one, entire sum is 1)

b) Sum is zero if and only if all are zero

Terms

(1) <u>literal</u>: b or $\bar{b}$ is a literal of b.

(2) <u>minterm</u>: product of literals for <u>all</u> boolean variables of f.

(3) <u>maxterm</u>: sum of literals for <u>all</u> boolean variables of f

ex) $f(x_1, x_2, x_3)$

focus on 1's — focus on 0's

x_1	x_2	x_3	minterms	maxterms
1	1	1	$x_1 x_2 x_3$	$\bar{x}_1 + \bar{x}_2 + \bar{x}_3$
1	1	0	$x_1 x_2 \bar{x}_3$	$\bar{x}_1 + \bar{x}_2 + x_3$
1	0	1	$x_1 \bar{x}_2 x_3$	$\bar{x}_1 + x_2 + \bar{x}_3$
1	0	0	$x_1 \bar{x}_2 \bar{x}_3$	$\bar{x}_1 + x_2 + x_3$
0	1	1	$\bar{x}_1 x_2 x_3$	$x_1 + \bar{x}_2 + \bar{x}_3$
0	1	0	$\bar{x}_1 x_2 \bar{x}_3$	$x_1 + \bar{x}_2 + x_3$
0	0	1	$\bar{x}_1 \bar{x}_2 x_3$	$x_1 + x_2 + \bar{x}_3$
0	0	0	$\bar{x}_1 \bar{x}_2 \bar{x}_3$	$x_1 + x_2 + x_3$

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ex)

x	y	f
1	1	1
1	0	0
0	1	0
0	0	1

Focus on 1's (minterms)

xy

$\bar{x}\bar{y}$

$$\boxed{f = xy + \bar{x}\bar{y}}$$

Sum of minterms = sum of products = minterm expansion
= disjunctive normal form

x	y	f
1	1	1
1	0	0
0	1	0
0	0	1

focus on 0's (maxterms)

$$\left.\begin{array}{l}\bar{x}+y\\ x+\bar{y}\end{array}\right\}\quad f=(\bar{x}+y)(x+\bar{y})$$

product of maxterms = product of sums = maxterm expansion
= conjunctive normal form

find function expressions

(1) table

Minterm expansion — focus on 1's

Maxterm expansion — focus on 0's

ex

x	y	z	f	
1	1	1	0	$\bar{x}+\bar{y}+\bar{z}$
1	1	0	0	$\bar{x}+\bar{y}+z$
1	0	1	1	$x\bar{y}z$
1	0	0	0	$\bar{x}+y+z$
0	1	1	0	$x+\bar{y}+\bar{z}$
0	1	0	1	$\bar{x}y\bar{z}$
0	0	1	1	$\bar{x}\bar{y}z$
0	0	0	0	$x+y+z$

$$f=(x\bar{y}z)+(\bar{x}y\bar{z})+(\bar{x}\bar{y}z)$$

$$f=(\bar{x}+\bar{y}+\bar{z})(\bar{x}+\bar{y}+z)(\bar{x}+y+z)(x+\bar{y}+\bar{z})(x+y+z)$$

(2) laws of boolean algebra

ex $f=(xy)+(yz)$

give minterm expansion $f=?$

$$f=(xy)+(yz)=(xy\cdot 1)+(1\cdot yz)$$
$$=(xy\cdot(z+\bar{z}))+((x+\bar{x})\cdot yz)$$
$$=(xyz)+(xy\bar{z})+(xyz)+(\bar{x}yz)$$

$$f = (xyz) + (xy\bar{z}) + (\bar{x}yz)$$

$$f = x + xz$$ maxterm expansion

$$f = x(x+z) = (x)(x+z)$$

$$f = (x+0)((x+z)+\boxed{0})$$
$$= (x+(y\cdot\bar{y}))((x+z)+(y\cdot\bar{y}))$$

$$f = (x+y)(x+\bar{y})(x+z+y)(x+z+\bar{y})$$
$$+0 \qquad +0$$

$$f = (x+y+z)(x+y+\bar{z})(x+\bar{y}+z)(x+\bar{y}+\bar{z})\cancel{(x+y+z)}\cancel{(x+\bar{y}+z)}$$

$$f = (x+y+z)(x+y+\bar{z})(x+\bar{y}+z)(x+\bar{y}+\bar{z})$$

$\boxed{\text{So}}$ **any** function can be expressed using variables and $\{+, \cdot, \bar{\ }\}$

← So we say this set of ops is functionally complete.

$\boxed{\text{Q}}$ Can we have a smaller set?

Consider DeMorgan's Law

$$\overline{x+y} = \bar{x}\cdot\bar{y} \rightarrow x+y = \overline{\bar{x}\cdot\bar{y}}$$

ex $f(x,y) = (\bar{x} \cdot y) + (x y) = \overline{\overline{(\bar{x} \cdot y)} \cdot \overline{(xy)}}$

So $\{\cdot, \bar{\ }\}$ is functionally complete b/c

$$x + y = \overline{\bar{x} \cdot \bar{y}}$$

But

$$\overline{x \cdot y} = \bar{x} + \bar{y} \rightarrow x \cdot y = \overline{\bar{x} + \bar{y}}$$

So $\{+, \bar{\ }\}$ is <u>also</u> functionally complete

ex $f(x,y) = (x+y)(\bar{x}+\bar{y})$

$$= \overline{\overline{(x+y)} + \overline{(\bar{x}+\bar{y})}}$$

<u>Consider:</u> NAND (not and)

x	y	$x \mid y$
1	1	0
1	0	1
0	1	1
0	0	1

NOR (not or)

x	y	$x \downarrow y$
1	1	0
1	0	0
0	1	0
0	0	1

$\bar{x} = x \mid x$

$x + y = (x \mid x) \mid (y \mid y)$

$x \cdot y = (x \mid y) \mid (x \mid y)$

So $\{\mid\}$ is functionally complete

$\bar{x} = x \downarrow x$

$x + y = (x \downarrow y) \downarrow (x \downarrow y)$

$x \cdot y = (x \downarrow x) \downarrow (y \downarrow y)$

So $\{\downarrow\}$ is functionally complete
