Math 322

Final Exam

Tues @ 1pm - 2^{50} pm (normal class room)

---

16 probs @ 10 pts each

140 pts = 100%

Exams 1 to 4 — "take" 4 problems each

ex

7) $R = \{(a,b),(c,d)\}$
a) sym closure?
b) ref. closure?

Note: "take"?

→ ① exact copy?

→ ② small changes? ex → $R = \{(a,c),(b,d),(c,a)\}$
a) sym closure?
b) reflexive closure?

→ ③ use same concepts? ex $R = \{(a,b) \mid a \geq b\}$
a) trans. closure?

Math 322 - Exam 1

1) Is the relation R consisting of all ordered pairs (a, b) such that a and b are people and have one common parent: reflexive, irreflexive, symmetric, antisymmetric, asymmetric, and/or transitive? If a property doesn't hold give a counter-example and **state the logical definitions of the properties** as you consider them.

2) Given the relation $R_1 = \{(a, b)|b = 2a\}$ on the set of positive integers from 1 to 12 and $R_2 = \{(a, b)|b = 3a\}$ find the relation $R_1 \cap R_2$.

3) The 4-tuples in a 4-ary relation represent these attributes of published books: title, ISBN, publication date, number of pages. What is the likely primary key for this relation? Under what conditions would (title, publication date) be a composite key?

4) Prove: If R on set A is transitive, then $\forall n R^n \subseteq R$, $n = 1, 2, 3, \ldots$

5) Represent the relation $R = \{(a, a), (a, c), (b, a), (c, a), (c, b)\}$ on the set $A = \{a, b, c, d\}$ as a digraph and a matrix.

6) For the set $A = \{a, b, c\}$, relation $R_1 = \{(a, a), (a, c), (b, b), (c, a)\}$, and relation $R_2 = \{(a, b), (b, a), (b, b), (c, c)\}$. Represent the relations as matrices and then use matrix operations to find $R_1 \circ R_2$.

closure

7) For $R = \{(a, a), (a, c), (a, d), (b, a), (b, d), (c, a), (c, d), (d, a), (d, c)\}$ on the set $A = \{a, b, c, d\}$ find the ...

a) Reflexive Closure as a matrix.

b) Symmetric Closure as a matrix.

8) For $R = \{(a, a), (a, b), (b, a), (b, c), (c, a)\}$ on the set $A = \{a, b, c\}$ find the transitive closure using the join of powers of M_R.

9) For $R = \{(a, a), (a, b), (a, d), (b, a), (b, c), (c, d), (d, c)\}$ on the set $A = \{a, b, c, d\}$ find the transitive closure using Warshall's Algorithm.

10) Show that the relation R consisting of all pairs (f, g) such that the second derivative of f and the second derivative of g are equal is an equivalence relation on the set of all polynomials with real-valued coefficients.

11) For the relation given above which functions are in the same equivalence class as $f(x) = 2x - 1$?

12) Show that (∷+, |) a partial ordering.

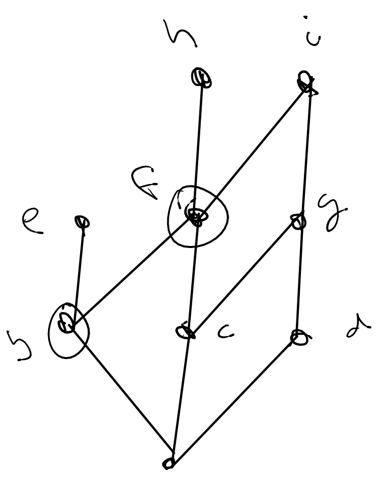
13) For the given Hasse diagram ...

a) State the maximal, minimal, greatest, and least elements.

b) Create a topological sort.

maximal: e, h, i   greatest: $\emptyset$

minimal: a   least: a



bounds of $\{b, f\}$

lower bounds: a, b   greatest of lower bounds: b

upper bounds: h, i, f   least of upper bounds: f

Math 322 - Exam 2

1a) Draw a multigraph with 5 vertices, 7 edges, and have exactly one vertex with degree 1.

1b) Draw a directed multigraph with 4 vertices, two of which have an in-degree of zero.

1c) Draw a mixed graph with 4 vertices.

1d) Draw a pseudograph with 5 vertices that has exactly two vertices of odd degree. Label those two vertices with o_1 and o_2.

2) Contruct the intersection graph for the sets for the universe of discourse being the integers from 1 to 10. The sets are $A = \{x | x > 5\}$, $B = \{x | x \text{ is even}\}$, $C = \{x | x \text{ is odd}\}$, $D = \{x | x \text{ is divisible by 2 or 3}\}$, and $E = \{x | x < 3\}$.

3) Draw the graph W_5 and state the number of vertices, edges, and degree for each vertex. Verify that the Handshake theorem applies.

4) Draw C_7 and determine if it is bipartite. Explain and name any theorems used to determine if it is, or is not, bipartite.

5) Are the graphs isomorphic? Justify your answer.

6) Are the graphs isomorphic? Justify your answer.

or undirected

7) Draw a directed multi-graph G_1 with 5 vertices that is weakly connected and not strongly connected. Draw a directed multi-graph G_2 with 4 vertices that is weakly connected and strongly connected.

8) For the given undirected graph find $\kappa(G)$ and $\lambda(G)$. State the vertices that make a minimal vertex cut. State the edges that make a minimal edge cut.

Euler

9) For the given puzzle, can you draw a continuous curve or a continuous closed curve that cuts each line segment exactly once? Explain your reasoning and any used theorems. If a curve exists draw one.

10) Can Dirac's and/or Ore's Theorems be applied to Q_3? Find a Hamilton Circuit for Q_3.

11) For what values of n will K_n have an Euler circuit? State the theorems you use.

12) Find the paths and lengths for the shortest paths between E and every other vertex in the graph.

MATH 322 - EXAM 3

1) You receive the following message via some social media application "Send the message 'I love to count in an advanced way' to 4 of your friends and you will get an A in Math 322!" If a total of 100 people send the message before it stops, how many people are in the tree? How many edges are in the tree? How many received it and did not send it out? What could you say about the height of the tree?

2) Prove: For an $m-$ary tree of height h with l leaves, $l \leq m^h$.

3) In a best case situation, how many weighings of a balance scale are needed if given four coins you may have a heavy counterfeit? Construct a decision tree to find the counterfeit or determine if there is no counterfeit.

4) Create a decision tree that orders the elements of the list a, b, c.

or tic-tac-toe

5) Draw the game tree for nim if the starting position consists of three piles with two, one, and one stones respectively [(2) , ~~(1)~~ , (1)]. Who wins the game if both players follow an optimal strategy?

6) Create the Huffman Code tree if a:25%, e:19%, i:18%, t:17%, s:14%, d:7% and encode "sad"

7) Write the inorder, preorder, and postorder traversal of the given tree.

8) For the standard expression $D_x[(2x+x^2)/(x+1)]$

a) Construct the rooted tree for the given expression.

b) Write the expression using post-fix notation.

c) Write the expression using pre-fix notation.

9) Use a bit table to verify De Morgan's laws $\overline{x+y} = \bar{x} \cdot \bar{y}$.

10) Using only the Identity, Complement, Associative, Communtative, and/or Distributive laws of a Boolean Algebra verify that $x \vee x = x$ and that $\bar{1} = 0$.

11) Find the sum of products for $F(x,y,z) = x \cdot (x + (y \cdot z))$ without using a table.

12) Find the product of sums for $F(x,y,z) = (x+y) \cdot z$ by using a table.

$\{\lambda, 1, 02, 0021, 000211, 00002111, \ldots\}$

$\lambda \cup 1 \cup \{0^{n+1} 2 1^n \mid n = 0, 1, 2, \ldots\}$

S; λ, 1, 0A; 02, 00A1; 0021, 000A11; 00021l, 0000A1''

MATH 322 - EXAM 4

1) For the grammar with $V = \{0, 1, 2, A, S\}$, $T = \{0, 1, 2\}$, and the productions $S \to 0A$, $S \to 1$, $S \to \lambda$, $A \to 0A1$, and $A \to 2$ find L(G).

? 6

2) Name the grammar type (give its type number and name) and circle the productions that prevent it from being the next type.

a) $S \to A, S \to B, S \to \lambda, A \to Ab, B \to aB, A \to a$, and $B \to b$

b) $S \to AB, A \to aAb, B \to bBa, A \to \lambda$, and $B \to \lambda$

c) $S \to ASB, S \to \lambda, B \to aAb, A \to a$, and $A \to B$

d) $S \to \lambda, S \to aA, A \to bB, B \to b$, and $A \to a$

3) Construct a finite-state machine with output that models a candy machine that accepts only pennies. Candy costs 3 cents and the machine always keeps the money for any amount greater than 3 cents. The customer can push buttons to receive candy or to return pennies. Represent the machine with a state table.

? 0 w $\boxed{\text{adder}}$

4) Construct a finite-state machine with output that delays input by two bits using 11 for the delay. Represent the machine with a state diagram.

5) Determine the language recognized by the given finite-state automaton.

2 0

6) Determine the language recognized by the given finite-state automaton.

2 0

7) Construct a deterministic finite-state automaton that recognizes the same language as the given non-deterministic finite-state automaton.

8) Using the constructions described in the proof of Kleene's Theorem, find a non-deterministic finite-state automaton that recognizes $(0 \cup 01)^*$.

8 6

9) Construct a non-deterministic finite-state automaton that recognizes the language generated by the regular grammar with $V = \{0, 1, A, S\}$, $T = \{0, 1\}$, and the productions $S \to 0A$, $S \to 1B$, $S \to \lambda$, $A \to 1A$, $A \to 1$, $B \to 0B$, $B \to 1$.

10) Let T be the Turing machine defined by the five-tuples: $(s_0, 0, s_1, 0, R)$, $(s_0, 1, s_1, 0, L)$, $(s_0, B, s_1, 1, R)$, $(s_1, 0, s_2, 1, R)$, $(s_1, 1, s_2, 1, R)$, and $(s_1, B, s_2, 1, R)$. Run the Turning machine on the below initial tape, write each of the positions, and determine the tape when T halts. Does T recognize the input string? What do you think T is doing?

? 0

Inital Tape: ... B,B,B,0,0,1,0,1,0,B,B,B, ...

11) Construct a Turning machine for the non-negative integers in unary format that computes the function $f(n_1, n_2) = n_1 + n_2$. Run your Turing machine on the input ... ,B,1,1,1,*,1,1,1,1,B ...

$(0 \cup 01)^*$