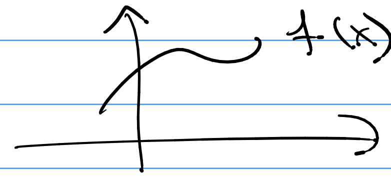
Math 344

Q's → geogebra.org

Calc 1,2 $\quad f: \mathbb{R} \to \mathbb{R}$



Calc 3 $\quad$ ch 13 $\quad f: \mathbb{R} \to \text{Vectors}$

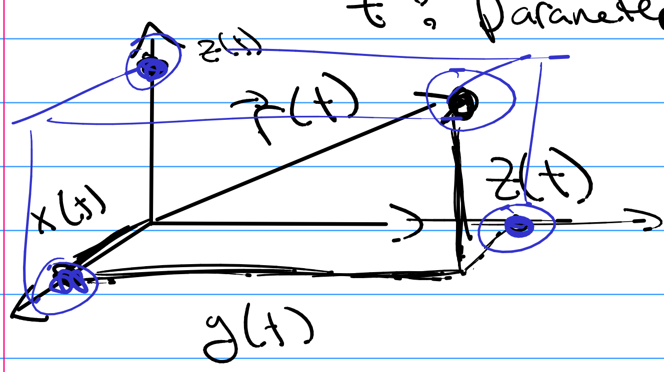
Change? Derivatives

Sum? Indef / def. Integrals

Represent $f: \mathbb{R} \to \text{Vectors}$

Notation: $\vec{r}(t) = \langle x(t), y(t), z(t) \rangle$

t : parameter



$x(t): \mathbb{R} \to \mathbb{R}$

(1) limits of $\vec{r}(t)$

$$\lim_{t \to a} \vec{r}(t) = \lim_{t \to a} \langle x(t), y(t), z(t) \rangle$$

So $\boxed{\lim_{t\to a} \vec{r}(t) = \left\langle \lim_{t\to a} x(t), \lim_{t\to a} y(t), \lim_{t\to a} z(t) \right\rangle}$

Ex $\vec{r}(t) = \left\langle \frac{\sin(t)}{t}, t^3 - t, \cos(t) \right\rangle$

$$\lim_{t\to 0} \vec{r}(t) = \left\langle \lim_{t\to 0} \frac{\sin(t)}{t}, \lim_{t\to 0} t^3 - t, \lim_{t\to 0} \cos(t) \right\rangle$$

$$= \langle 1, 0, 1 \rangle$$

If $\lim_{t\to a} \vec{r}(t) = \vec{r}(a)$, $\vec{r}(t)$ is continuous

---

with limits and continuity we can talk about

1. "change" → Derivatives
2. "Sums" → Def. Integrals

Visualize (See movie)

Space Curve

parametric eq'n's of C

C is $\begin{cases} x = x(t) \\ y = y(t) \\ z = z(t) \end{cases}$ t: parameter

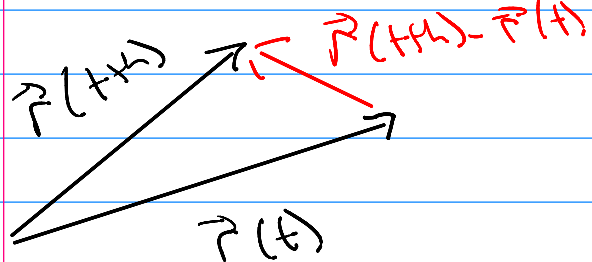
$$\vec{r}(t) = \langle \underline{x(t)}, \underline{y(t)}, \underline{z(t)} \rangle$$

C is $\vec{r}(t)$'s tip traced out over
$t \in [\text{start}, \text{end}]$

Deriv.

Calc 1, 2 $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

Calc 3 $\vec{r}\,'(t) = \lim_{h \to 0} \boxed{\frac{\vec{r}(t+h) - \vec{r}(t)}{h}}$



How to Calculate?

$$\lim_{h \to 0} \frac{\vec{r}(t+h) - \vec{r}(t)}{h} = \lim_{h \to 0} \frac{\langle x(t+h), y(t+h), z(t+h)\rangle - \langle x(t), y(t), z(t)\rangle}{h}$$

$$= \lim_{h \to 0} \left\langle \frac{x(t+h) - x(t)}{h}, \frac{y(t+h) - y(t)}{h}, \frac{z(t+h) - z(t)}{h} \right\rangle$$

So $\boxed{D_t[\vec{r}(t)] = \langle x'(t), y'(t), z'(t) \rangle}$

and $\boxed{\int \vec{r}(t)\,dt = \left\langle \int x(t)\,dt, \int y(t)\,dt, \int z(t)\,dt \right\rangle + \vec{C}}$

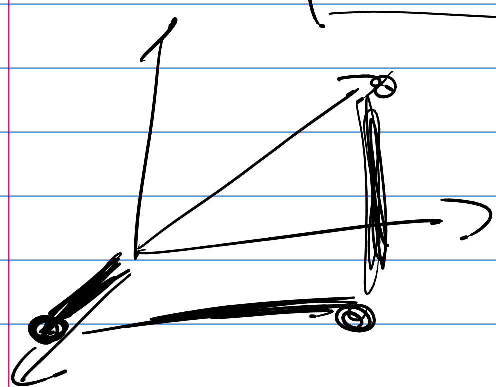
$$\int_a^b \vec{r}(t)\,dt = \lim_{n\to+\infty}\left(\sum_{i=1}^{n} \vec{r}(t_i^*)\right)\Delta t \qquad \Delta t = \frac{b-a}{n}$$

$$= \lim_{n\to+\infty} \left\langle \underline{\sum_{i=1}^{n} x(t_i^*)\Delta t},\ \underline{\sum_{i=1}^{n} y(t_i^*)\Delta t},\ \underline{\sum_{i=1}^{n} z(t_i^*)\Delta t} \right\rangle$$

$$\boxed{\int_a^b \vec{r}(t)\,dt = \left\langle \int_a^b x(t)\,dt,\ \int_a^b y(t)\,dt,\ \int_a^b z(t)\,dt \right\rangle}$$

What can we do?

$$D_t[\vec{r}(t)],\quad \int \vec{r}(t)\,dt,\quad \int_a^b \vec{r}(t)\,dt$$



$$\vec{r}(t) = \langle x(t), y(t), z(t)\rangle$$

Properties & Derivative

$$[\vec{u} + \vec{v}]' = \vec{u}' + \vec{v}'$$

$$[c\,\vec{u}(t)]' = c\,\vec{u}'(t)$$

$$[f(t)\,\vec{u}(t)]' = f'(t)\,\vec{u}(t) + f(t)\,\vec{u}'(t)$$

$$[\vec{u} \cdot \vec{v}]' = \vec{u}' \cdot \vec{v} + \vec{u} \cdot \vec{v}'$$

$$[\vec{u} \times \vec{v}]' = \vec{u}' \times \vec{v} + \vec{u} \times \vec{v}'$$