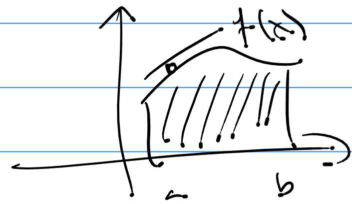
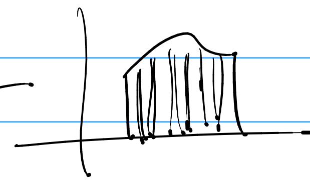
Math 344

Meaning? 2D

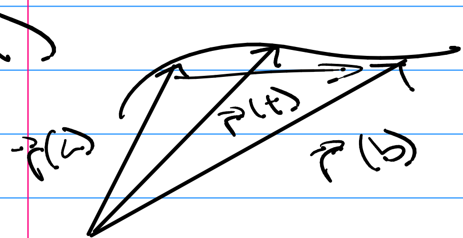


$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$ $\longrightarrow$ if f is velocity, f' is change in vel = accel

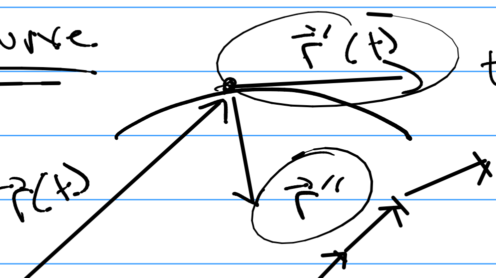
$$\int_a^b f(x)\,dx = \text{area}$$ —  $\sum f(x_i^*)\Delta x$

$$\int_a^b \underset{\text{accel}}{f'(x)}\,dx = \underset{\text{vel @ } b}{\underline{f(b)}} - \underset{\text{vel @ } a}{\underline{f(a)}}$$

$\vec{r}(t)$

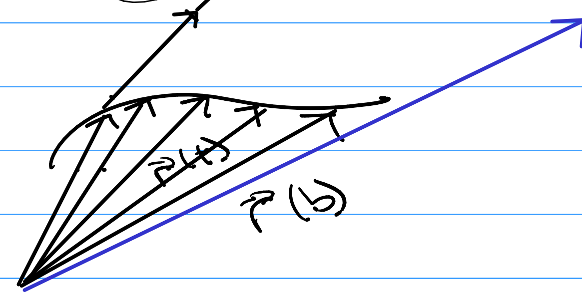


Space curve



tangent to space curve

$$\int_a^b \underline{\vec{r}(t)\,dt} = \lim_{n\to\infty} \sum_{i=1}^{n} \vec{r}(t_i^*)\,\Delta t$$

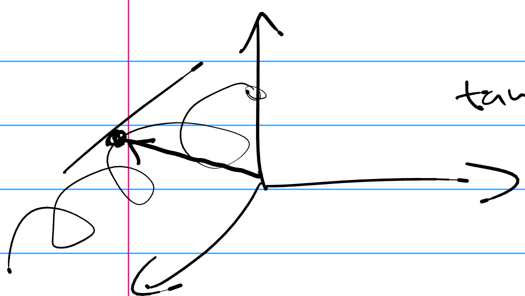


$$\int_a^b \vec{r}'(t)\,dt = \vec{r}(b) - \vec{r}(a)$$

Apps ① tangent lines — need: ① point

② vector that is in direction to line

(ex) $\vec{r}(t) = \langle t, \cos(t^2), 3+\sin(2t)\rangle$



tangent line when $t = 1$

point: $\vec{r}(1) = \langle 1, \cos(1), 3+\sin(2)\rangle$

Vector in tangent direction: $\vec{r}\,'(1)$

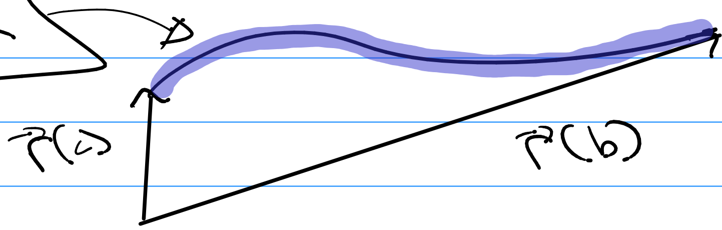
$$\vec{r}\,' = \langle 1, -2t\sin(t^2), 2\cos(2t)\rangle$$

$$\vec{r}\,'(1) = \langle 1, -2\sin(1), 2\cos(2)\rangle$$

tangent line: parametric form:

$$x = 1+t$$
$$y = \cos(1) - 2\sin(1)\,t$$
$$z = 3+\sin(2) + 2\cos(2)\,t$$

Arclength

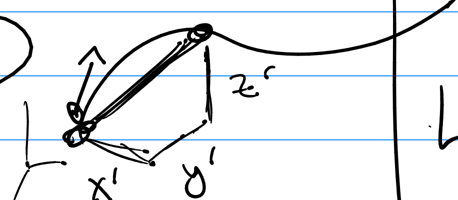


$\vec{r}(t)$ is parametric

(2D)

$$L = \int_a^b \sqrt{(x'(t))^2 + (y'(t))^2}\,dt$$

(3D)



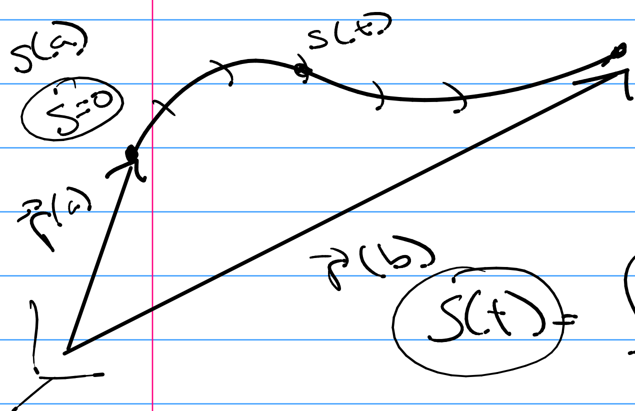
$$\boxed{L = \int_a^b \sqrt{(x')^2 + (y')^2 + (z')^2}\,dt}$$

Vector Notation:

$$L = \int_a^b |\vec{r}\,'|\,dt$$

ex $\quad \vec{r}\,' = \langle 1, -2t\sin(t^2), 2\cos(2t) \rangle$

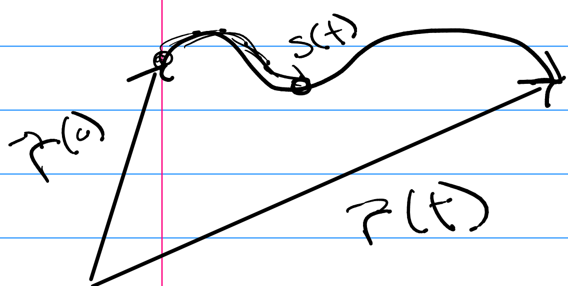
from 0 to 1 $\quad L = \int_0^1 \left(1 + 4t^2\sin^2(t^2) + 4\cos^2(2t)\right)^{1/2} dt$

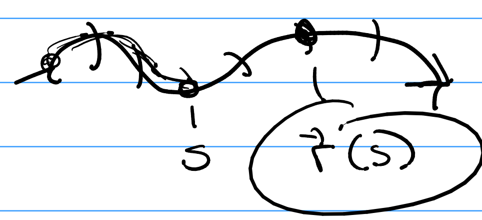


$s(t)$ = arc length function

$$S(t) = \int_a^t |\vec{r}\,'(u)|\,du = \int_a^t \sqrt{(x'(u))^2 + (y'(u))^2 + (z'(u))^2}\,du$$

$$\frac{ds}{dt} = s'(t) = |\vec{r}\,'(t)|$$





find

$$\vec{r}(t) \rightsquigarrow \boxed{t = t(s)} \to \vec{r}(t(s))$$

do ① $S(t) = \boxed{\int_a^t |\vec{r}\,'(u)|\,du}$ = expression with t's

so S = expression of t's

algebra solve for t

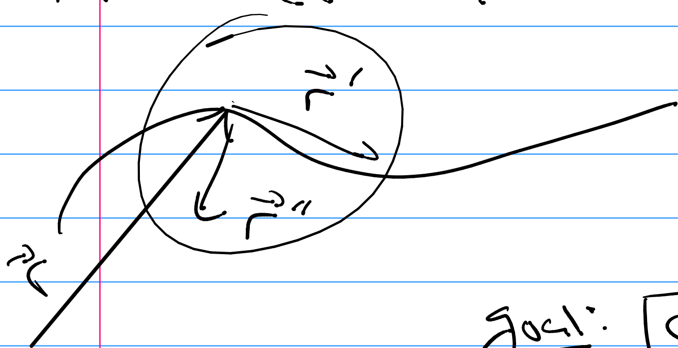
t = expression of s's = $t(s)$

② plug in $t(s)$ into $\vec{r}(t)$

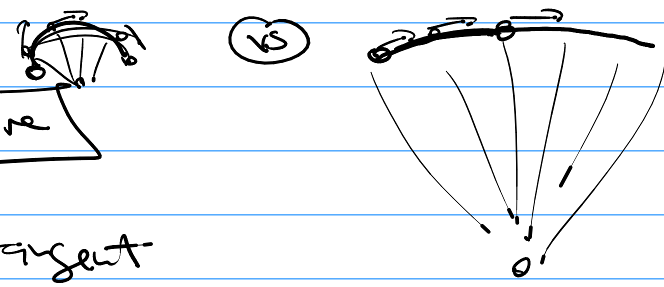
$$\boxed{\therefore \ \vec{r}(t(s)) = \vec{r}(s)}$$

parameterization of $\vec{r}$ with respect to arc length.

Metrics (Vectors / Scalars) to describe the curve.



① Measure to tell between



goal: $\boxed{\text{Curvature}}$

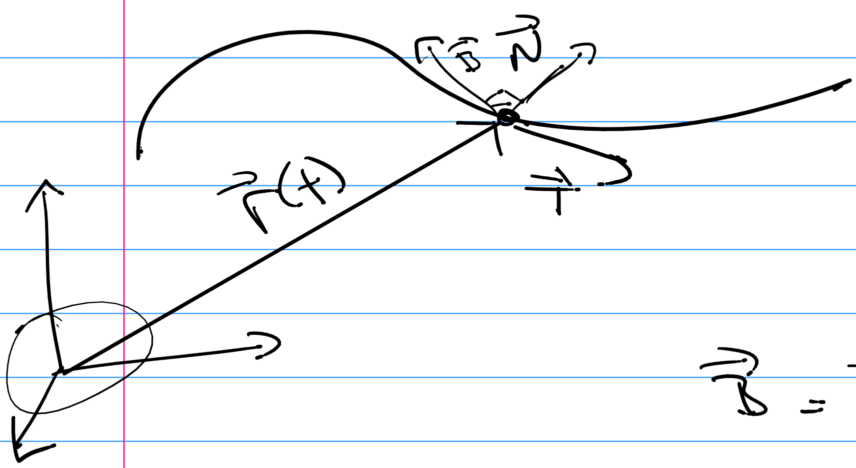
a) $\vec{T}(t) = \dfrac{\vec{r}'(s)}{|\vec{r}'|}$ unit tangent

b) define curvature $\boxed{\kappa = \left|\dfrac{d\vec{T}}{ds}\right|}$

Consider: $\dfrac{d\vec{T}}{ds} = \dfrac{d\vec{T}/dt}{ds/dt} = \dfrac{\vec{T}'(t)}{|\vec{r}'(t)|}$

So $\boxed{\kappa = \left|\dfrac{\vec{T}'(t)}{\vec{r}'(t)}\right|}$

Unit Tangent, Unit Normal, Unit Binormal



$$\vec{T} = \frac{\vec{r}'}{|\vec{r}'|}$$

$$\vec{N} = \frac{\vec{T}'}{|\vec{T}'|}$$

$$\vec{B} = \vec{T} \times \vec{N}$$