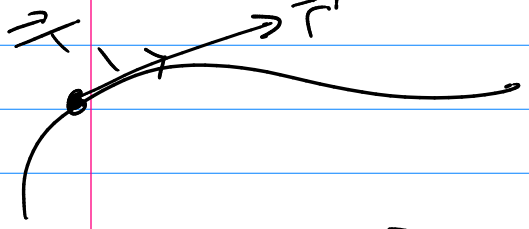
Math 344

Q's 13.3

$\vec{r}(t) = \langle x(t), y(t), z(t) \rangle \to \vec{r}', \vec{r}''$

$$L = \int_a^b \sqrt{(x'(t))^2 + (y'(t))^2 + (z'(t))^2}\, dt$$

$$\to s(t) = \int_a^t |\vec{r}'(u)|\, du$$



$$\vec{T} = \frac{\vec{r}'}{|\vec{r}'|} = \frac{1}{|\vec{r}'|}\vec{r}'$$

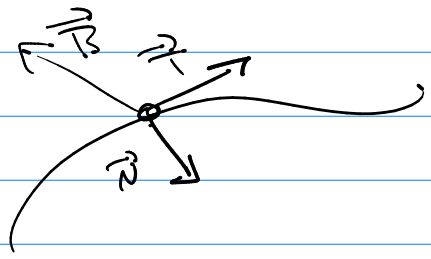
Ex $\vec{r} = \langle t, 3t^2, \sin(t) \rangle$

$\vec{r}' = \langle 1, 6t, \cos(t) \rangle$

$|\vec{r}'| = \sqrt{1 + 36t^2 + \cos^2(t)}$

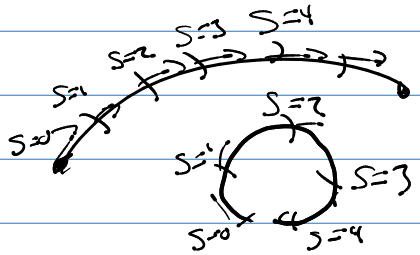
$$\vec{T}(t) = \frac{1}{\sqrt{1+36t^2+\cos^2(t)}} \langle 1, 6t, \cos(t) \rangle$$

$$\vec{N}(t) = \frac{\vec{T}'(t)}{|\vec{T}'(t)|}$$



$$\vec{B} = \vec{T} \times \vec{N}$$

$$\kappa = \left| \frac{d\vec{T}}{ds} \right| = \frac{|\vec{T}'(t)|}{|\vec{r}'(t)|}$$



$$K = \frac{|\vec{r}' \times \vec{r}''|}{|\vec{r}'|^3}$$

13.4 Motion in 3D Space

Vector Equation of Motion

$\vec{r}(t)$ → $\vec{r}'(t) = \vec{v}(t)$ → $\vec{r}''(t) = \vec{v}'(t) = \vec{a}(t)$

position → velocity → acceleration

$a(t)$ → $\int a(t)\,dt = \vec{v}(t) + C$ → $\int \vec{v}(t)\,dt = \vec{r}(t) + C$

acceleration

need an initial value to find

need an initial value.

Note: speed $= |\vec{v}(t)| = v$ (def.)

Apps

① Newton's Second Law & Motion.

we can measure this → $\vec{F}(t) = \frac{d}{dt}\{m \cdot \vec{v}(t)\} = m\,\vec{a}(t)$

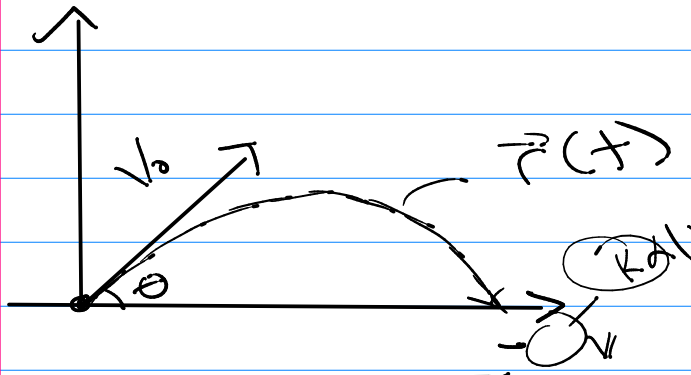
$$\vec{a} = \frac{\vec{F}(t)}{m}$$

$$\vec{v} = \int \vec{a}\,dt$$

$$\vec{r} = \int \vec{v}\,dt$$

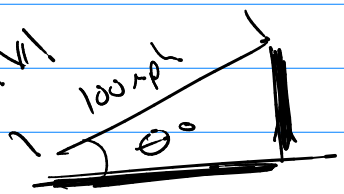
Find eqn's of motion.

ex projectile motion (ignore air)



$\vec{r}(0) = \langle 0, 0 \rangle$

$\vec{v}(0) = \langle 100\cos\theta, 100\sin\theta \rangle$



$a_x = K|v|v$

$\propto v^2$

$\vec{a} = \langle 0, -g \rangle$ (forces)

$\vec{v}(t) = \int \langle 0, -g \rangle \, dt = \langle 0, -gt \rangle + \vec{c}$

when $\vec{v}(0) = \langle 100\cos\theta, 100\sin\theta \rangle$

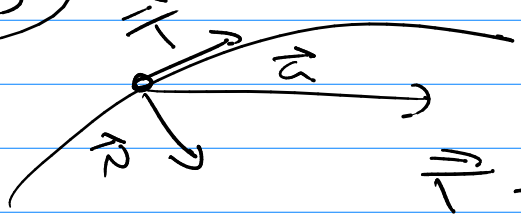
$\vec{v}(t) = \langle 0, -gt \rangle + \langle 100\cos\theta, 100\sin\theta \rangle$

$\vec{v}(t) = \langle 100\cos\theta, 100\sin\theta - gt \rangle$

$\vec{r}(t) = \langle 100\cos\theta\, t, 100\sin\theta\, t - \frac{1}{2}gt^2 \rangle + \vec{c}$

$\vec{0}$

App



$\vec{a} = a_T \vec{T} + a_N \vec{N}$

$\vec{T} = \frac{\vec{v}}{v}$ so $\vec{v} = v\vec{T}$

$\vec{a} = \vec{v}' = v'\vec{T} + v\vec{T}'$

↓ calc + alg. Next class

Goal $\vec{a} = \boxed{v'}\vec{T} + \boxed{\kappa v^2}\vec{N}$

a_T a_N