Math 344

Exam 1

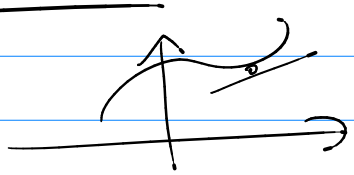
Monday take home exam (due Wed)

12.5, 12.6
13.1 – 13.4
} 2 probs / section

Webassign homework will start back up this weekend ...

Calculus: → study functions

calc 1, 2 $f: \mathbb{R} \to \mathbb{R}$



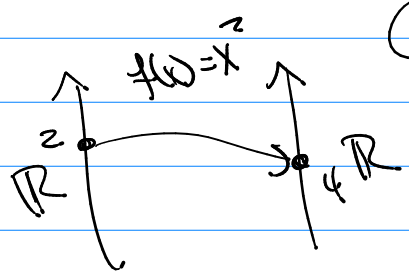
$f(x)$
$f'(x)$
$\int_a^b f(x)\,dx$

calc 3 → ch 13 → Vector functions: $f: \mathbb{R} \to$ Vector.

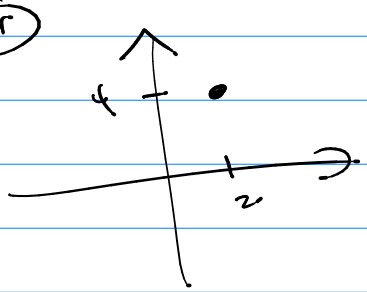
ch 14 New function types to ... $f: \mathbb{R}^n \to \mathbb{R}$

a) limit?
b) continuity?
c) change?
d) sum?

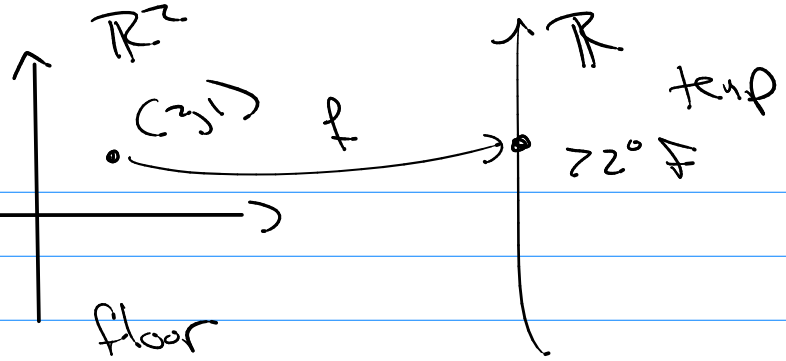
ex $f: \mathbb{R} \to \mathbb{R}$

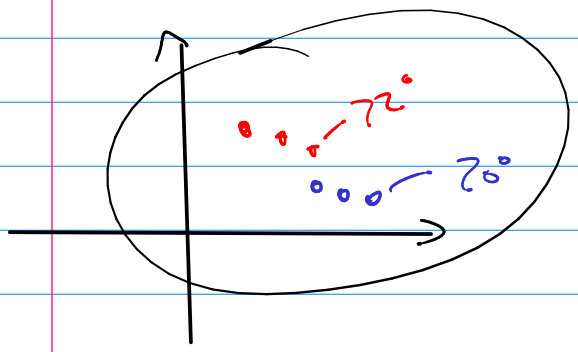


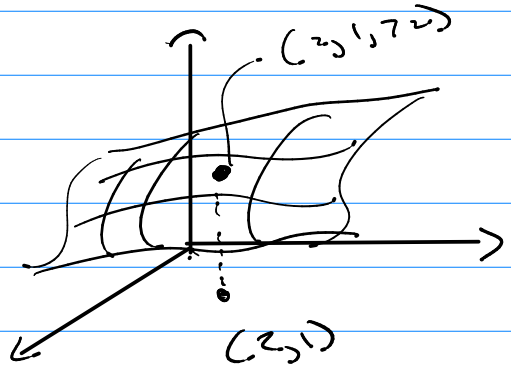
or



$f: \mathbb{R}^2 \to \mathbb{R}$







$f: \mathbb{R}^n \to \mathbb{R}$ (ex) 12.6

Implicit (vs) Explicit

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$$

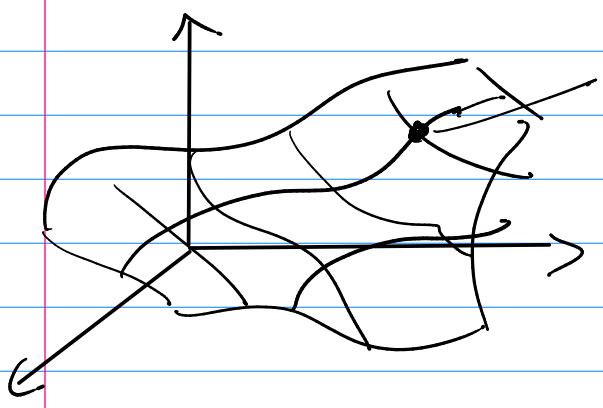
$z = f(x,y)$

$$z = \frac{x^2}{a^2} - \frac{y^2}{b^2} = f(x,y)$$

$z = f(x,y)$

Graph

$f: \mathbb{R}^2 \to \mathbb{R}$ $\quad z = f(x,y)$



How to graph?

① known? ($\mathbb{R}^3$ objects)
lines, spheres, planes, conics.

② use traces / level curves to draw it.

$z = f(x,y)$ → pick several z : $\{z_0, z_1, z_2, z_3, \dots\}$
(set constants.)

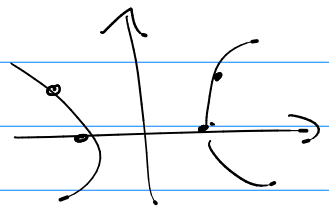
ex $z = \frac{x^2}{4} - y^2$

level curves: let $z = -2, -1, 0, 1, 2$

$z = 0$ $\quad 0 = \frac{x^2}{4} - y^2 \rightarrow y^2 = \frac{x^2}{4}$

$y = \frac{1}{2}|x|$

$z = 1$ $\quad 1 = -\frac{x^2}{4} + y^2$



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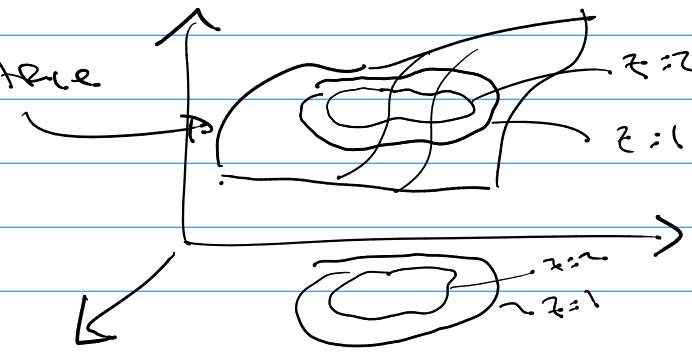
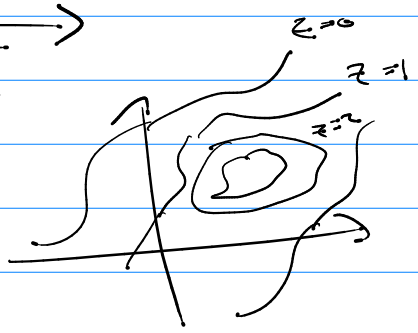
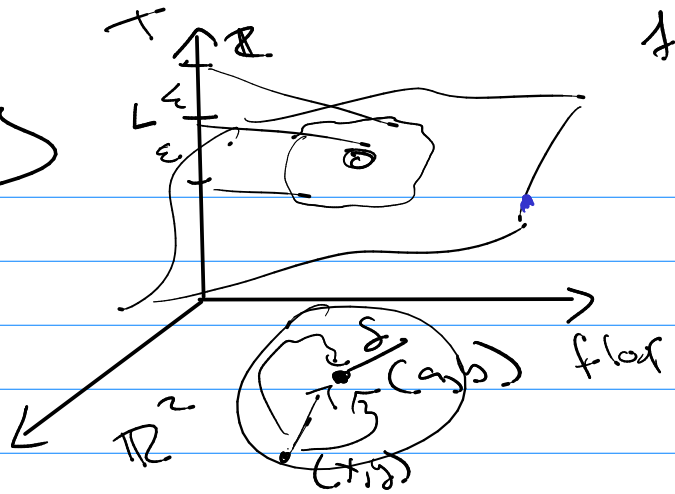
level curves as trace



level curves on domain = contour plot

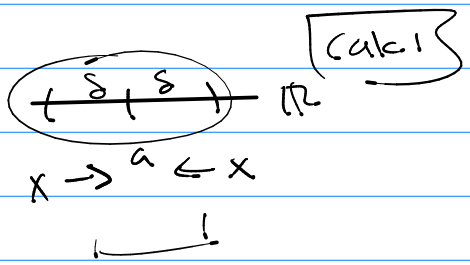


14.2



$f: \mathbb{R}^2 \to \mathbb{R}$

$z = f(x,y)$



Def $\lim_{(x,y)\to(a,b)} f(x,y) = L$

for all $\varepsilon > 0$, there is a $\delta > 0$ such that for (x,y) in domain of $f(x,y)$

"when (x,y) are with δ of (a,b)" then "$f(x,y)$ is within ε of L"

$\mathbb{R}^2$ $\quad 0 < \sqrt{(x-a)^2 + (y-b)^2} < \delta \to |f(x,y) - L| < \varepsilon$

$\mathbb{R}^n$ Vector notation $\quad 0 < |x - a| < \delta \to |f(x) - L| < \varepsilon$

a ↑ point

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typical problem is when does $\lim_{(x,y)} f(x,y)$ not exist?

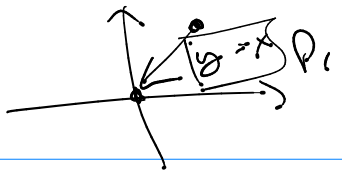
if $(x,y) \to (a,b)$ along path P_1 get L_1 as limit

if $(x,y) \to (a,b)$ along path P_2 get L_2 as limit.

$L_1 \neq L_2$ no limit

Ex

$$\lim_{(x,y)\to(0,0)} \frac{5y^4\cos^2 x}{x^4+y^4}$$



along a path $\rightarrow$ on P_1

$$\lim_{x\to 0} \frac{5x^4\cos^2 x}{2x^4} = \frac{5}{2}\lim_{x\to 0}\cos^2 x$$

$$= 5/2 = L_1$$