Math 344

Q's Note: Exam 1 → Do it yourself!

ch4 $f: \mathbb{R}^n \to \mathbb{R}$

ex $f: \mathbb{R}^2 \to \mathbb{R}$ $f(x,y) = x^3 - y^2 + \sin(xy)$

ex $f: \mathbb{R}^4 \to \mathbb{R}$ $T(a,b,c,d) = \sin(ab) - \cos(cd)$

B limits, continuity

Def: $\lim_{\mathbf{x} \to \mathbf{a}} f(\mathbf{x}) = L$

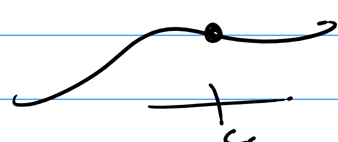
for all $\varepsilon > 0$ there is a $\delta > 0$

if $0 < |\mathbf{x} - \mathbf{a}| < \delta \to |f(\mathbf{x}) - L| < \varepsilon$

Continuity f is cont.

$$\lim_{\mathbf{x} \to \mathbf{a}} f(\mathbf{x}) = f(\mathbf{a})$$

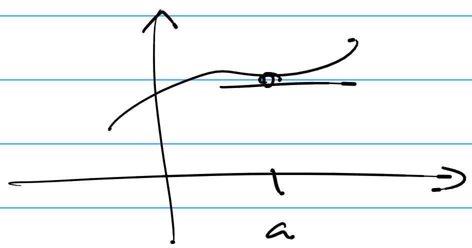
ex $\lim_{x \to 4} \boxed{2x^2 - 1} = 2(4)^2 - 1 = 31$



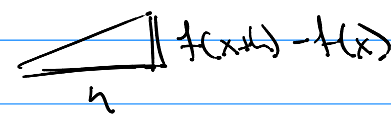
Similar

$$\lim_{(x,y)\to(1,2)} (3x^2 - y^3 + xy) = 3(1)^2 - (2)^3 + (1)(2)$$
$$= \boxed{-3}$$

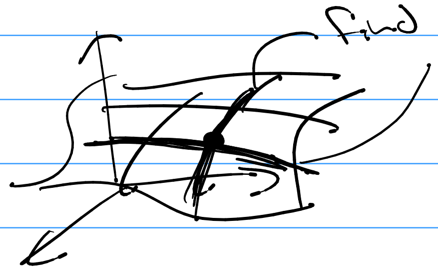
Change? **Calc 1** $f: \mathbb{R} \to \mathbb{R}$



$$\lim_{h\to 0} \frac{f(a+h)-f(a)}{h} = f'(a)$$



Calc 3 ex $f: \mathbb{R}^2 \to \mathbb{R}$



find 'derivative' along a trace (hold one variable constant)

→ <u>Partial derivative.</u>

Partial Derivatives @ **point (a,b)**

<u>hold</u> y - constant, deriv. with respect to x.

$$f_x(a,b) = \lim_{h\to 0} \frac{f(a+h,b) - f(a,b)}{h}$$

<u>hold</u> x - constant, deriv. with respect to y

$$f_y(a,b) = \lim_{h\to 0} \frac{f(a,b+h) - f(a,b)}{h}$$

@ (x,y) $$f_x(x,y) = \lim_{h\to 0} \frac{f(x+h,y) - f(x,y)}{h}$$

$$f_y(x,y) = \lim_{h\to 0} \frac{f(x,y+h) - f(x,y)}{h}$$

Note: Calc 1,2 $y = f(x)$ y - dep. variable

$$\frac{d}{dx}[y^3] = 3y^2 \cdot \frac{dy}{dx}$$

Now? $z = f(x,y)$ x,y are ind. variable

z is the dep. variable

ex $f(x,y) = x^3 + y^3$ } b/c y is a const. accord. to partial with respect to x.

$f_x = 3x^2 + 0$

$f(t,s) = t^3 + s^3$

$f_t = 3t^2 + 0 = 3t^2$

Partial Derivative Notation. $f(v_1, v_2, v_3, v_4)$

$$\boxed{f_t} = \boxed{\frac{\partial}{\partial t}(f) = \frac{\partial f}{\partial t}} = D_1 f = D_t f$$

ex $f(s,t,u,v) = \sin(st) + \underline{\cos(uv)} - stuv^2$

$f_s = \boxed{t\cos(st) + 0 - tuv^2}$ ← is a function as well.

$f_t = s\cos(st) + 0 - suv^2$ (take partial's again)

$f_u = 0 - v\sin(uv) - stv^2$

$f_v = 0 - u\sin(uv) - 2stuv$

Higher Order Partial Derivatives

ex $f: \mathbb{R}^2 \to \mathbb{R}$ $\quad T = f(u,v)$

$T(u,v) = \sin(uv) - u^2\sqrt{v}$

1st order $\quad T_u, T_v$

2nd order $\quad T_{uu}, T_{uv}, T_{vu}, T_{vv}$

ex $T_u = v\cos(uv) - 2u\sqrt{v}$

$T_v = u\cos(uv) - \underline{\frac{1}{2}u^2 v^{-1/2}}$

$T_{uu} = -v^2\sin(uv) - 2\sqrt{v}$

$T_{uv} = \cos(uv) - uv\sin(uv) - uv^{-1/2}$

$T_{vv} = -u^2\sin(uv) + \frac{1}{4}u^2 v^{-3/2}$

$T_{vu} = \cos(uv) - uv\sin(uv) - uv^{-1/2}$

$\boxed{\text{Thm (Clairaut's) } f_{xy} = f_{yx} \text{ if } f_{xy}, f_{yx} \text{ are cont. on a disk containing } (a,b).}$

Partial Differential Equations.

$$f_x + f_y = 3\cos(xy)$$

Solve? $\to f(x,y)$