Math 344

Qs

$$\frac{\cosh(t)}{\cosh^2(t)-1} \to \frac{\cosh(t)}{\sinh^2(t)} \to \frac{1}{u^2}\,du$$

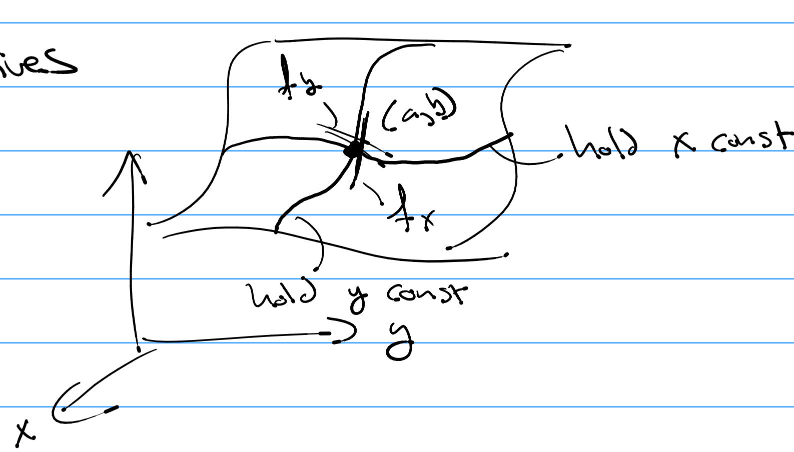
6.1 $\cosh^2 x - \sinh^2 x = 1$

$u = \sinh(t)$

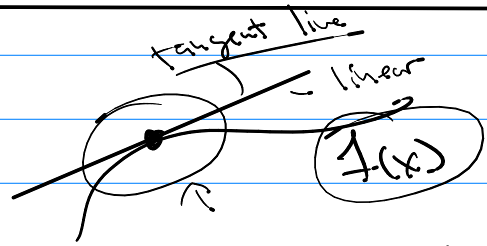
$du = \cosh(t)\,dt$

14.3 Partial Derivatives

$z = f(x,y)$



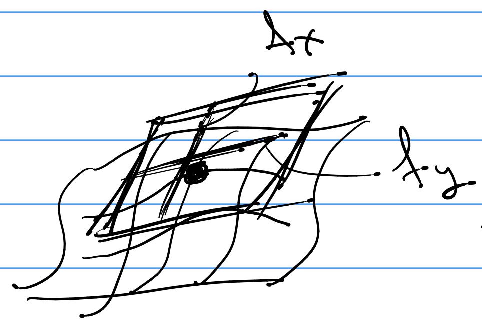
Calc 1



need cont. no corners

$z = T(x,y)$

$f(x,y)$



tangent plane

contains tangent lines based on f_x, f_y

eqn of a plane @ (x_0, y_0, z_0)

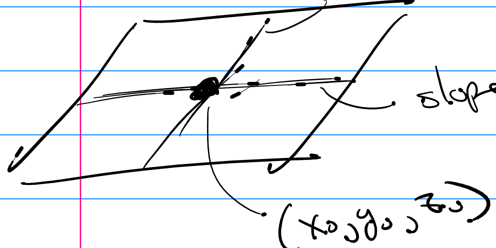
$$\frac{A}{c}(x-x_0) + \frac{B}{c}(y-y_0) + (z-z_0) = 0$$

$a =$ $\quad$ $b =$

$\hookrightarrow$ solve for z

$$z = z_0 + a(x - x_0) + b(y - y_0)$$

slope holding $y = y_0$ change w.r.t. x = $f_x(x_0, y_0)$



slope holding $x = x_0$ change w.r.t. y = $f_y(x_0, y_0)$

but if $x = x_0$

plane: $z = z_0 + 0 + b(y - y_0)$

$$z = z_0 + b(y - y_0)$$

So $b = f_y(x_0, y_0)$

similarly $a = f_x(x_0, y_0)$

$\therefore$ tangent plane

$$\boxed{z = z_0 + f_x(x_0, y_0)(x - x_0) + f_y(x_0, y_0)(y - y_0)}$$

ex $\boxed{z = 2x^2 + y^2 - 5y \quad @ \quad (\overset{x_0}{1}, \overset{y_0}{2}, \overset{z_0}{-4})}$

tangent plane: $z = z_0 + f_x(x_0, y_0)(x - x_0) + f_y(x_0, y_0)(y - y_0)$

$f_x = 4x \quad , \quad f_y = 2y - 5$

$$\boxed{z = -4 + 4(x - 1) + (-1)(y - 2)}$$

Application:

① near (x_0, y_0, z_0) $\quad f(x,y) \approx$ tangent plane

(local linear approximation)

$$2x^2 + y^2 - 5y \approx -4 + 4(x-1) + (-1)(y-2)$$

$$2x^2 + y^2 - 5y \approx \boxed{4x - y - 6} \quad \text{near } (1, 2, -4)$$

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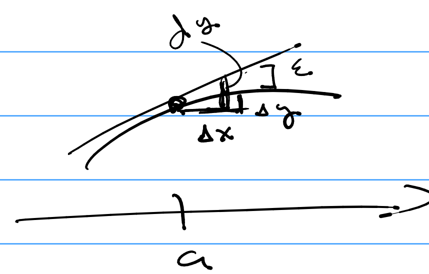
use this tangent plane idea to extend def. of f being differentiable.

$\boxed{\text{Calc 1}}$ $\quad f$ is differentiable @ $x = a$

if

$$\frac{\Delta y}{\Delta x} = f'(a) + \varepsilon$$



$$\boxed{\Delta y = f'(a)\Delta x + \varepsilon \Delta x}$$

as $\Delta x \to 0$, $\varepsilon \to 0$

$z = f(x,y)$, f is differentiable @ (a,b)

if $\quad \Delta z = f_x(a,b)\,\Delta x + f_y(a,b)\,\Delta y + \varepsilon_1 \Delta x + \varepsilon_2 \Delta y$

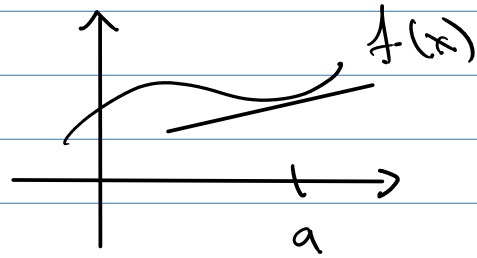
as $\Delta x \to 0$, $\Delta y \to 0$ then $\varepsilon_1 \to 0$, $\varepsilon_2 \to 0$

$\boxed{\text{th}^{\text{n}}}$ if f_x, f_y exist near (a,b) and are cont @ (a,b)

then f is differentiable @ (a,b)

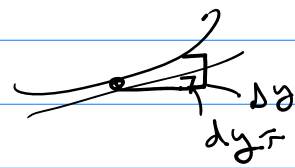
Application: error propagation.

$\boxed{\text{Calc 1}}$



Measure $x = a \pm \Delta x$ (Δx = dx, error)

$$\Delta y \approx dy = f'(x)\Delta x$$



$\boxed{\text{Calc 3}}$ total differential $dz = f_x(x,y)\,dx + f_y(x,y)\,dy$

@ (a,b)

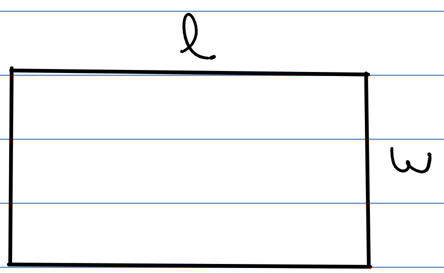
$$dz = f_x(a,b)(x-a) + f_y(a,b)(y-b)$$

tangent plane:

$$f(x,y) \approx f(a,b) + dz$$

and for errors $\Delta z \approx dz$

ex



$l = 30\text{ cm} \pm 0.1\text{ cm}$

$w = 20\text{ cm} \pm 0.2\text{ cm}$

$$A = \boxed{l \cdot w} = 30 \cdot 20 = \boxed{600\text{ cm}^2}$$

error in $A = \Delta A \approx dA = f_w \Delta w + f_\ell \Delta \ell$

$$\Delta A \approx \ell \cdot \Delta w + w \Delta \ell = (30)(.2) + (20)(.1)$$

$$\Delta A \approx 6 + 2 = 8 \text{ cm}^2$$

$$\boxed{A = 600 \text{ cm}^2 \pm 8 \text{ cm}^2}$$