Math 344

Q's

$\underbrace{g(x,y)}_{z} = (36 - x^2 - y^2)^{1/2}$ $\qquad z = 0, 4, 5, 6$

$\qquad z = k$

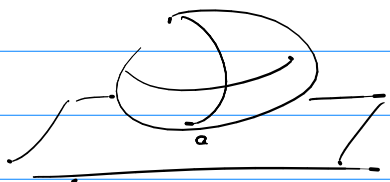
$z^2 = 36 - x^2 - y^2$

$x^2 + y^2 + z^2 = 6^2$ $\qquad$ sphere center $= (0,0,0)$

$r = 6$

$x^2 + y^2 = \boxed{(6^2 - k^2)}$ $\qquad$ circle center $(0,0)$

$r = (6^2 - k^2)^{1/2}$



K

14.5 Chain Rule for partial derivatives

and Implicit Partial Derivatives.

chain rule

Calc1 $\frac{d}{dx}\left[\cos(x^2)\right] = -\sin(x^2)\cdot(2x)$

$$\frac{d}{dx}\left[f(g(x))\right] = f'(g(x))\cdot g'(x)$$

Calc3 $f : \mathbb{R}^n \to \mathbb{R}$

(ex) $f : \mathbb{R}^2 \to \mathbb{R}$ $\qquad z = f(x,y)$

Composition: (ex) $x(t)$, $y(t)$

$$z = f(x(t), y(t)) = f(t)$$

ex $\boxed{f(x,y) = x^2 + y^2, \text{ but } x(t) = t^3 - 1, \; y(t) = \sin(t^2+1)}$

$$\boxed{f(x(t), y(t)) = (t^3-1)^2 + \sin^2(t^2+1) = f(t)}$$

Note: b/c $z = f(t)$ then $\dfrac{dz}{dt}$ is defined.

ex
$$\frac{dz}{dt} = 2(t^3-1)^1(3t^2) + 2\sin(t^2+1)\cos(t^2+1)\,(2t)$$

chain rule of partial deriv. $f(x,y)$, $x(t)$, $y(t)$

$$\frac{dz}{dt} = \frac{\partial f}{\partial x}\frac{dx}{dt} + \frac{\partial f}{\partial y}\frac{dy}{dt}$$

ex $f = x^2 + y^2$, $x(t) = (t^3-1)$, $y(t) = \sin(t^2+1)$

$$\boxed{\frac{dz}{dt} = (2x)(3t^2) + (2y)\cos(t^2+1)(2t)}$$

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ex $f(x,y)$, $x(s,t)$, $y(s,t)$

so $z = f(x(s,t), y(s,t)) = f(s,t)$

therefore: $z_s = ?$, $z_t = ?$

chain rule: $\dfrac{\partial z}{\partial s} = \dfrac{\partial z}{\partial x}\cdot\dfrac{\partial x}{\partial s} + \dfrac{\partial z}{\partial y}\cdot\dfrac{\partial y}{\partial s}$; $\dfrac{\partial z}{\partial t} = \dfrac{\partial z}{\partial x}\dfrac{\partial x}{\partial t} + \dfrac{\partial z}{\partial y}\dfrac{\partial y}{\partial t}$

Sub notation:

$$z_s = z_x x_s + z_y y_s$$

$$z_t = z_x x_t + z_y y_t$$

Ex $f(x,y) = \sin(xy) + x$

$x(t_1,t_2) = t_1^2 + t_1 t_2$, $y(t_1,t_2) = \sin(t_1) + t_2$

So $f_{t_1} = f_x x_{t_1} + f_y y_{t_1}$

$$\boxed{f_{t_1} = (y\cos(xy)+1)(2t_1+t_2) + (x\cos(xy))(\cos(t_1))}$$

$$f_{t_2} = f_x x_{t_2} + f_y y_{t_2}$$

$$\boxed{f_{t_2} = (y\cos(xy)+1)(t_1) + (x\cos(xy))(1)}$$

Ex $f(x_1,x_2,x_3,\dots,x_n)$ $f: \mathbb{R}^n \to \mathbb{R}$

Let: $x_1(t_1,t_2,\dots,t_m)$

$x_2(t_1,t_2,\dots,t_n)$

$\vdots$

So $f(t_1,t_2,\dots,t_n)$

we can find $\left(\frac{\partial f}{\partial t_1}, \frac{\partial f}{\partial t_2}, \dots\right)$

Chain Rule

$$\frac{\partial f}{\partial t_1} = \frac{\partial f}{\partial x_1}\cdot\frac{\partial x_1}{\partial t_1} + \frac{\partial f}{\partial x_2}\frac{\partial x_2}{\partial t_1} + \dots + \frac{\partial f}{\partial x_n}\frac{\partial x_n}{\partial t_1}$$

$$f_{t_2} = f_{x_1} x_{1_{t_2}} + f_{x_2} x_{2_{t_2}} + \dots + f_{x_n} x_{n_{t_2}}$$

$\boxed{\text{Implicit Function Th}^m}$ (Implicit Partial Derivatives)

ex $\boxed{z = \sqrt{36-x^2-y^2}} \rightarrow \boxed{x^2+y^2+z^2-36} = 0$

z = explicit function of x, y's

$z = f(x,y)$

$F(x,y,z) = 0$

implicit function for problem

$z = f(x,y)$

b/c z is a function of x, y (implicit rep of explicit rep)

$\therefore$ z_x, z_y can be found.

ex $z = \sqrt{36-x^2-y^2}$

explicit partial $\quad z_x = \frac{1}{2}(36-x^2-y^2)^{-1/2}(-2x)$

Q: can we use $x^2+y^2+z^2-36=0$ (implicit form) to find z_x?

Use chain rule: $\quad F(x,y,z) = 0$

$\frac{\partial}{\partial x}$ of both sides $\quad \frac{\partial F}{\partial x}\cdot\underbrace{\boxed{\frac{\partial x}{\partial x}}}_{=1} + \frac{\partial F}{\partial y}\cdot\underbrace{\boxed{\frac{\partial y}{\partial x}}}_{=0} + \frac{\partial F}{\partial z}\boxed{\frac{\partial z}{\partial x}} = 0$

$\to \quad \dfrac{\partial z}{\partial x} = - \dfrac{\frac{\partial f}{\partial x}}{\frac{\partial f}{\partial z}}$ or $\boxed{z_x = -\dfrac{f_x}{f_z}}$

Similarly $\boxed{z_y = -\dfrac{f_y}{f_z}}$

ex $\underbrace{\boxed{x^2 + y^2 + z^2 - 36}}_{F} = 0$

$$\boxed{\begin{aligned} &\to z_x = -\frac{2x}{2z} = -\frac{x}{z} \\ &\to z_y = -\frac{2y}{2z} = -\frac{y}{z} \end{aligned}}$$