Math 394

$\boxed{Q15}$ $F(x,y) = \int_y^x \cos(t^6)\,dt$

F_x , F_y

Fund. thm

a) $\int_a^b f(x)\,dx = F(b) - F(a)$

$$\frac{d}{dx}F(x) = f(x)$$

b) $F(x) = \int_a^x f(t)\,dt$

where $F(a) = 0$ and $\frac{d}{dx}[F(x)] = f(x)$

? $F(x,y) = \int_y^x \cos(t^6)\,dt$? F_x ?

consider:

$$F(x) = \int_0^x \cos(t^6)\,dt$$

Calc 1 idea

$$\frac{d}{dx}F(x) = \cos(x^6)$$

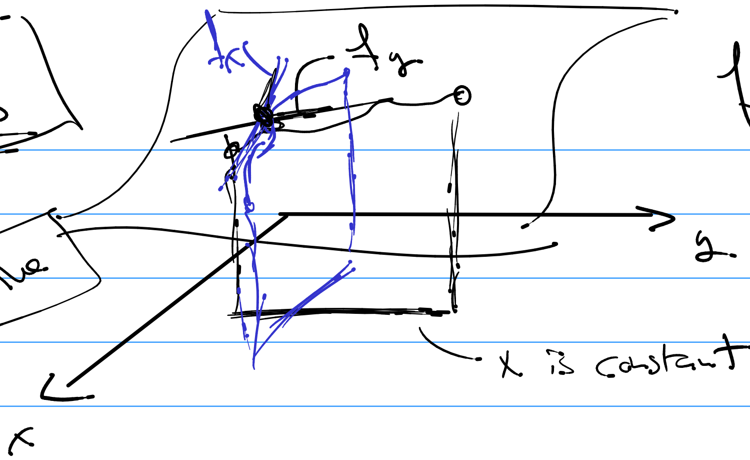
by Fund. thm $F_x = \cos(x^6)$

$$F_y = -\cos(y^6)$$

$\vec{i} = \langle 1,0\rangle$

Partials

partial Derivative

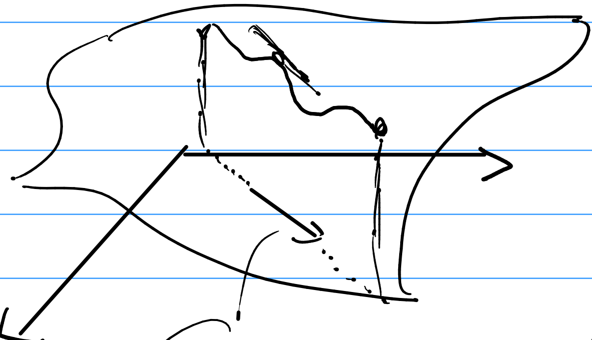


f_x change in x-direction (y is const)

$\vec{j} = \langle 0,1\rangle$

f_y is change in y-direction (x is const)

Q



$f(x,y)$

$\vec{u}$ — can we have a definition for "change" in direction of $\vec{u}$

Def Directional Derivative of $f(x,y)$ @ (x_0,y_0)

in direction of unit vector $\vec{u} = \langle a,b\rangle$

$$D_{\vec{u}} f(x_0,y_0) = \lim_{h\to 0} \frac{f(x_0+ah,\, y_0+bh) - f(x_0,y_0)}{h}$$

Note: $D_{\langle 1,0\rangle} f = f_x$

$D_{\langle 0,1\rangle} f = f_y$

Thm If $f(x,y)$ is differentiable

then $D_{\vec{u}} f(x,y) = f_x a + f_y b$

Quick vector notation

$$D_{\vec{u}} f(x,y) = \langle f_x, f_y \rangle \cdot \langle a,b \rangle$$

or $D_{\vec{u}} f(x,y) = \langle f_x, f_y \rangle \cdot \vec{u}$

seems this might be useful.

Move up to $f: \mathbb{R}^3 \to \mathbb{R}$ $\quad f(x,y,z)$

do same study as above...

$$D_{\vec{u}} f(x,y,z) = \lim_{h \to 0} \frac{f(\vec{x}_0 + h\vec{u}) - f(\vec{x}_0)}{h}$$

Thm $D_{\vec{u}} f = \langle f_x, f_y, f_z \rangle \cdot \vec{u}$

Def gradient of $f(x,y)$ or $f(x,y,z)$

$$\text{grad}(f(x,y)) = \nabla f(x,y) = \langle f_x, f_y \rangle$$

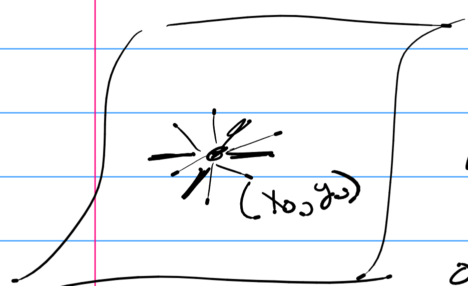
$$\text{grad}(f(x,y,z)) = \nabla f(x,y,z) = \langle f_x, f_y, f_z \rangle$$

Now $\boxed{D_{\vec{u}} f = \nabla f \cdot \vec{u}}$ $\quad f(x,y)$, $\quad f(x,y,z)$

Q does ∇f have anything special about it?

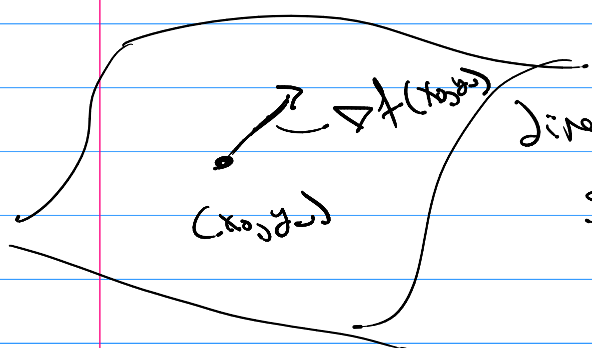
Thm if you maximize the value of $\boxed{D_{\vec{u}} f}$

(find $\vec{u}$ such that $D_{\vec{u}} f$ is max)



Maximum is $|\nabla f|$

occurs when $\vec{u}$ is a direction of ∇f.



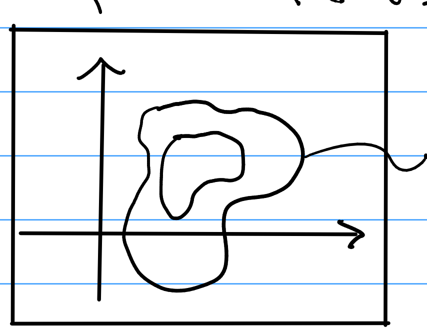
direction of max. ascent

steepness (slope) $|\nabla f|$

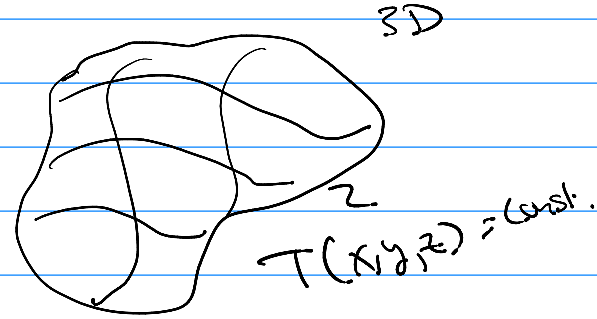
∇f and level curves $(f(x,y) = \text{const})$

and level surfaces $(f(x,y,z) = \text{const})$

Idea plate $T(x,y)$



3D



Chain rule

level curve

$$f(x(t), y(t)) = \text{const}$$

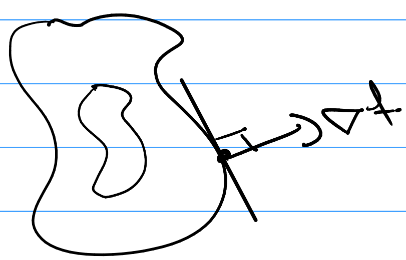
$$\underline{f_x \frac{dx}{dt} + f_y \frac{dy}{dt} = 0}$$

$$\langle f_x, f_y \rangle \cdot \langle x', y' \rangle = 0$$

$$\nabla f \cdot \vec{r}\,' = 0$$

$\vec{r}$ <u>curve</u>

$\vec{r}\,'$ <u>tangent line</u>.



level surface.

$$f(x(t), y(t), z(t)) = \text{const}$$

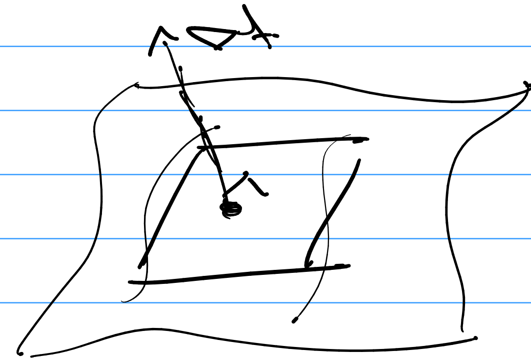
$$\underline{f_x \frac{dx}{dt} + f_y \frac{dy}{dt} + f_z \frac{dz}{dt} = 0}$$

$$\langle f_x, f_y, f_z \rangle \cdot \langle x', y', z' \rangle = 0$$

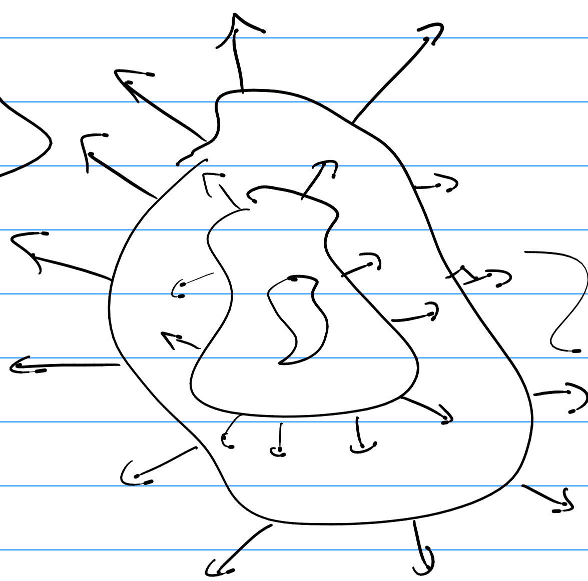
$$\nabla f \cdot \vec{r}\,' = 0$$

$\vec{r}$ surface

$\vec{r}\,'$ tangent <u>plane</u>.



Summary



$\langle f_x, f_y \rangle$

grad f $\langle f_x, f_y, f_z \rangle$