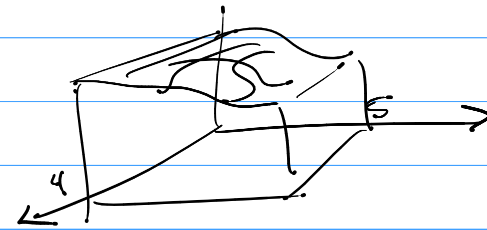
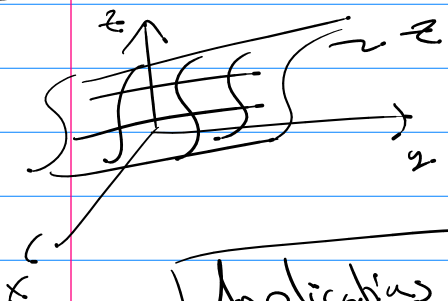
Math 344

Q's $f(x,y) = 4x + 6y - x^2 - y^2 + 3$

abs. extrema



14.7 $f: \mathbb{R}^2 \to \mathbb{R}$



$z = f(x,y)$

ch 14 "change" mean for $f(x,y)$

→ partials, mixed partials

→ directional derivatives

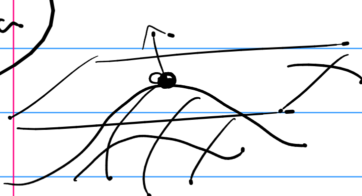
Applications ① differentials (error estimates)

② chain rule

14.7 high spots / low spots on surface

maximum — minimum

Idea



tangent plane is "flat"

Def $f(x,y)$ has a local max @ (a,b)

for all (x,y) $f(x,y) \le f(a,b)$ (x,y) around (a,b)

and local max value is $M = f(a,b)$

$f(x,y)$ has a local min @ (a,b)

for all (x,y) $f(x,y) \geq f(a,b)$ (x,y) around (a,b)

and local min value is $m = f(a,b)$

---

if inequalities hold for all (x,y) in domain

→ $\boxed{\text{absolute max / min}}$

$\boxed{\text{Step 1}}$ Where to look for local extrema?

Idea: possible local extrema if tangent plane is flat there.

$\boxed{\text{thm}}$ if f does have a local max/min @ (a,b) then

$$f_x(a,b) = f_y(a,b) = 0$$

$\boxed{\text{So}}$ $\boxed{\text{find}}$ critical points (a,b) such that $f_x(a,b) = 0$, $f_y(a,b) = 0$

(a,b)

ex $f(x,y) = 4x + 6y - x^2 - y^2 + 3$

$f_x = 4 - 2x$ $\quad f_x(a,b) = 0 \to 4 - 2a = 0$

$f_y = 6 - 2y$ $\quad f_y(a,b) = 0 \to 6 - 2b = 0$

$a = 2 \quad b = 3$

$\boxed{\text{So}}$ $(2,3)$ is a critical point + we may have an extrema here.

2nd partials $f_{xx}, f_{xy}, f_{yx}, f_{yy}$

Calc I $f'(c)=0$

$f''(c)$ <0, >0, $=0$

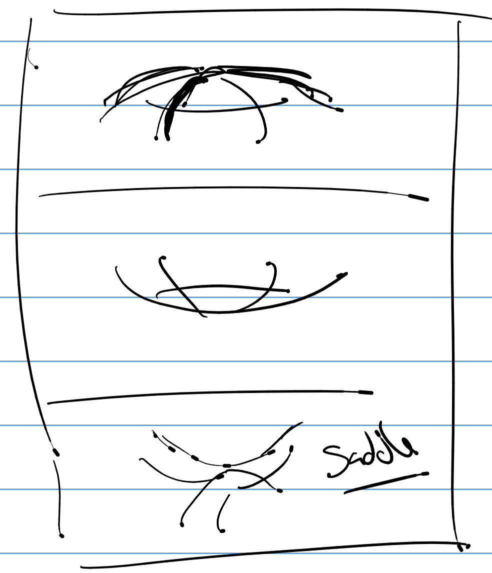
2nd Derivatives Test

$f_{xx}, f_{xy}=f_{yx}, f_{yy}$ are cont around (a,b) and (a,b) is a critical point

f_{xx}

f_{yy}

$f_{yx}=f_{xy}$



let

$$D = f_{xx}f_{yy} - (f_{xy})^2$$

① $D>0$ and $f_{xx}<0$ → local max

② $D>0$ and $f_{xx}>0$ → local min

② $D<0$ → not a local max/min (saddle point)

also $D = \begin{vmatrix} f_{xx} & f_{xy} \\ f_{yx} & f_{yy} \end{vmatrix}$

ex $f(x,y) = 4x+6y-x^2-y^2+3$

$f_x = 4-2x$
$f_y = 6-2y$ → critical point $(2,3)$

$D = f_{xx}f_{yy} - (f_{xy})^2$

$D(2,3) = 4-0$
$= 4$

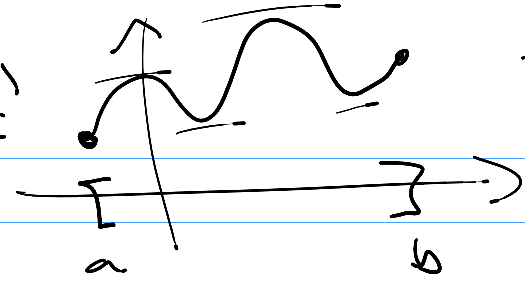
$f_{xx} = -2$ $f_{xy} = 0$

$f_{yx} = 0$ $f_{yy} = -2$

∴ local max @ $(2,3)$ of $f(2,3) = 16$

abs. extrema

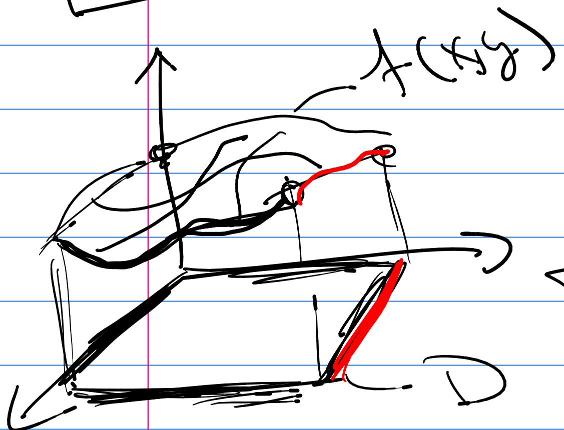
Calc1



$f(x)$ on $[a,b]$

→ have both abs. max and min @ local extrema or end points (boundry)

Calc3



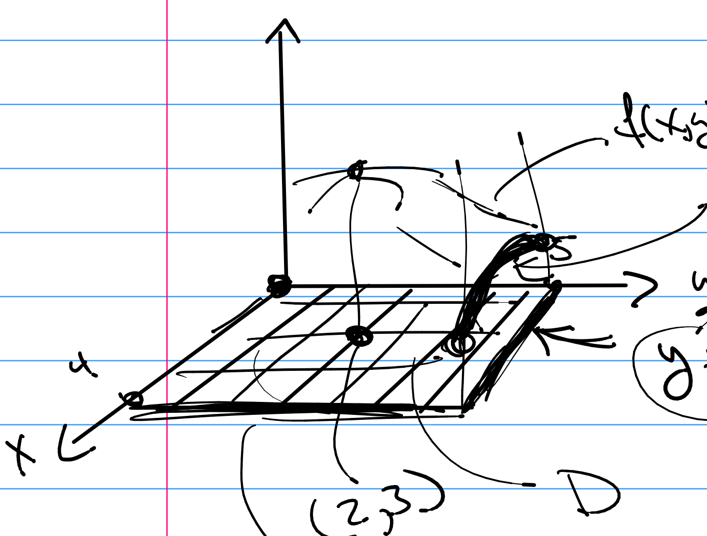
Thm: f is cont. on closed and bounded domain D

then f has abs max and abs. min. @ local extrema or boundry.

ex $f(x,y) = 4x + 6y - x^2 - y^2 + 3$

→ local max of 16 @ $(2,3)$

$D: 0 \le x \le 4$ and $0 \le y \le 5$



$f(x,y) = \overset{16}{4x} + 6y \overset{-16}{-x^2} - y^2 + 3$

$f(x,5) = 4x - x^2 + 8 \quad 0 \le x \le 4$

$y = 5$

$f(4,y) = 6y - y^2 + 3 \quad 0 \le y \le 5$