Math 344

Q's $\boxed{\text{Exam 2}}$

→ Monday : Review & 15.1 Lecture

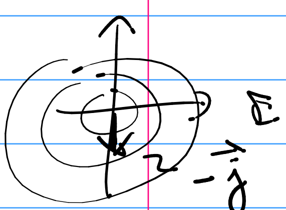
→ Wed. $\boxed{\text{in-class}}$ Exam 2

Q's 14.6 #34

dep.

$\boxed{z} = 100 - \frac{x^2}{200} - \frac{y^2}{100}$ $\mathbb{R}^2 \to \mathbb{R}$

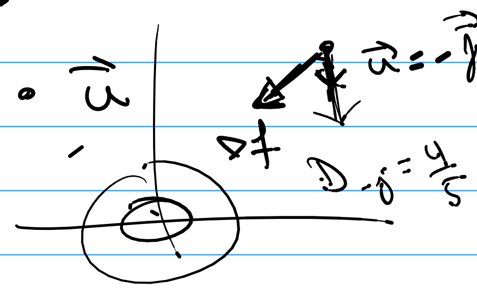
$P = (60, 40, 966)$



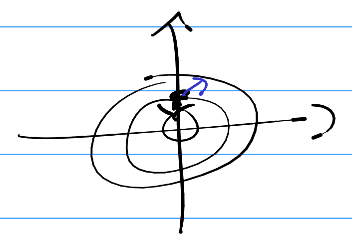
a) $D_{(-\vec{j})} f = \nabla f \cdot (\langle 0, -1 \rangle)$

$D_{\vec{u}} f = \langle -\frac{x}{100}, -\frac{y}{50} \rangle \cdot \vec{u}$ $(60,40)$

@ $(60,40)$ $D_{\vec{u}} f = \langle -\frac{3}{5}, -\frac{4}{5} \rangle \cdot \vec{u}$



$\nabla f = \langle -\frac{x}{100}, -\frac{y}{50} \rangle$

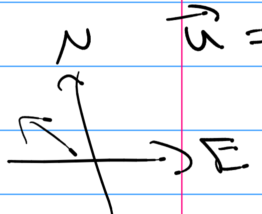


$\nabla f(0,1) = \langle 0, -\frac{1}{50} \rangle$

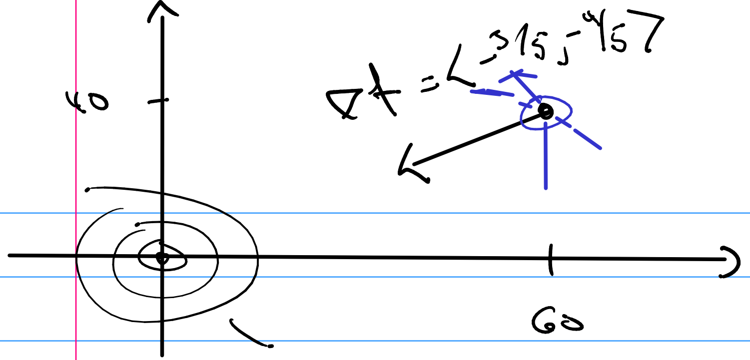
---

36 $D_{\vec{u}} f = \langle -\frac{x}{100}, -\frac{y}{50} \rangle \cdot \vec{u}$

$\vec{u} = (NW) = \langle -1/\sqrt{2}, 1/\sqrt{2} \rangle$



@ $(60,40)$ $\langle -\frac{3}{5}, -\frac{4}{5} \rangle \cdot \langle -1/\sqrt{2}, 1/\sqrt{2} \rangle$



$\boxed{14.7}$ Max / Min $z = f(x,y)$

① critical solve: $f_x = 0$, $f_y = 0$ $\xrightarrow{\text{solve}} (a,b)$ critical points

② 2nd Derivative Test

$$D = \boxed{f_{xx} f_{yy}} - (f_{xy})^2$$

$D > 0$ → and $f_{xx} < 0$ (a,b) has a max

→ and $f_{xx} > 0$ (a,b) has a min

$D < 0$ (a,b) has a saddle point

or

③ f given a fixed boundry domain

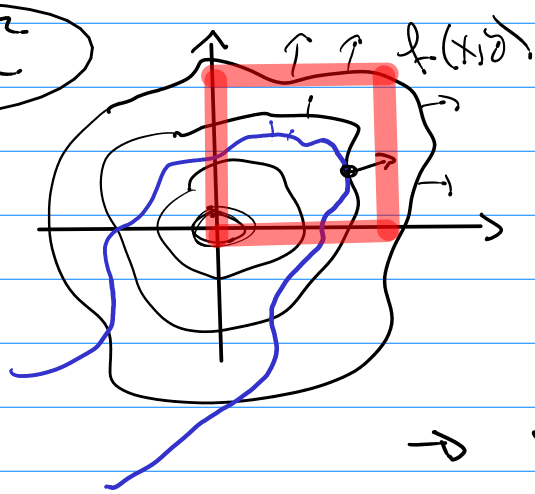
→ find criticals, find max/min's on boundry → largest = max
smallest = min

14.8 $f(x,y,z)$ is a function we want to maximize.

→ placed under a constraint eqn. $\boxed{g(x,y,z) = K}$

ex in $\mathbb{R}^2$

$f(x,y)$



Notice max f walk on g is when ∇f is parallel to ∇g

→ $\nabla f = (\text{const}) \nabla g$

Method of Lagrange Multipliers

Maximize f with respect to one constraint function $g = K$

(assume extrema exist and $\nabla g \neq \vec{0}$ on $g = K$)

step 1 Solve the system of eqn's

$\nabla f = \lambda \nabla g$ → $f_x = \lambda g_x$, $f_y = \lambda g_y$, $f_z = \lambda g_z$ → solve find (x,y,z,λ)

and

$g = K$ → $g = K$

Step 2 evaluate $f(x,y,z)$ → largest = max, smallest = min

$\boxed{\text{2 constraints}}$ Max/Min f

constraints: $g = k_1$, $h = k_2$

Step 1 Solve system of eqn's

$\nabla f = \lambda \nabla g + \mu \nabla h$

and constraints

$$\begin{cases} f_x = \lambda g_x + \mu h_x \\ f_y = \lambda g_y + \mu h_y \\ f_z = \lambda g_z + \mu h_z \\ g = k_1 \\ h = k_2 \end{cases}$$

Solve → (x, y, z, λ, μ)