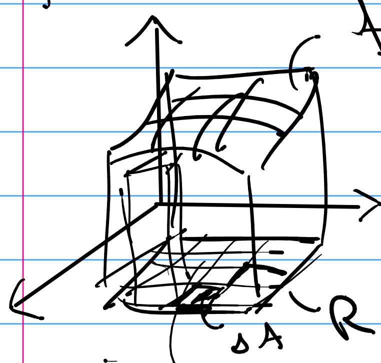
Math 344

$f: \mathbb{R}^n \to \mathbb{R}$

ex. $f: \mathbb{R}^2 \to \mathbb{R}$



ch 14 $\boxed{\text{Change}}$ $f_x, f_y, f_{xx},$ $D_u f$ ∇f

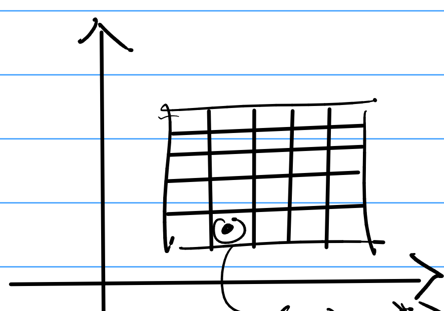
$\boxed{\text{Ch15}}$ $\boxed{\text{Sums}}$ "Integration"

$\boxed{15.1}$ Find Volume

$$\iint_R f(x,y)\,dA = \lim_{m,n\to\infty} \sum_{i=1}^{m}\sum_{j=1}^{n} f(x_{ij}^*, y_{ij}^*)\,\Delta A$$

approximate:
$$\iint_R f(x,y)\,dA \approx \sum_{i=1}^{m}\sum_{j=1}^{n} f(x_{ij}^*, y_{ij}^*)\,\Delta A$$

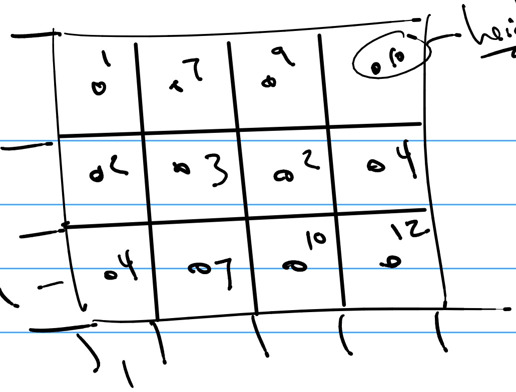
techniques: picking (x_{ij}^*, y_{ij}^*)



(x_{ij}^*, y_{ij}^*) of the grid. height $= f(x_{ij}^*, y_{ij}^*)$

midpt approx: $\Delta A\big(f(\text{midpt}_1) + f(\text{midpt}_2) + \cdots + \big)$

Ex



height = f(midpt)

$\Delta A = 0.28$

$\Delta y = 1.4$

$\Delta x = 0.2$

$$Vol \approx 0.28(1+7+9+10+2+3+2+4+4+7+10+12)$$

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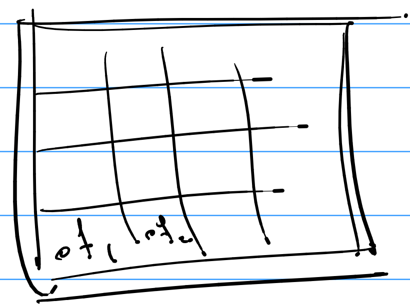
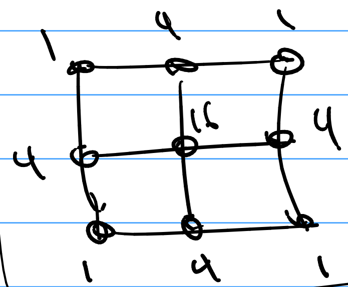
or



Area $= A$

$$Vol \approx A \cdot (\text{ave } f\text{'s})$$



$$V_s = A(\text{weighted ave})$$

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if given $f(x,y)$ as a function..

$$\iint_R f(x,y)\, dA = ?$$

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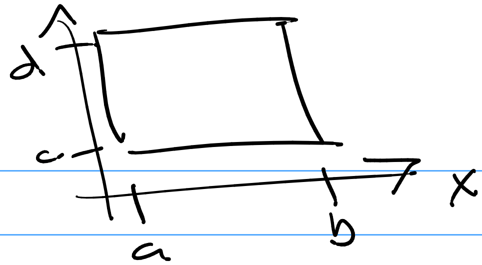
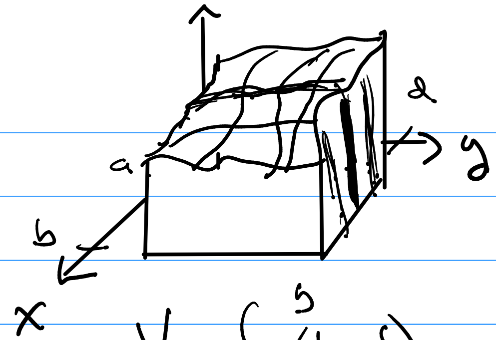
calc idea

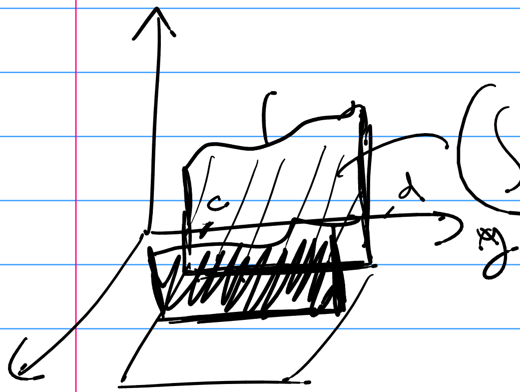
$$\int_a^b f(x)\,dx = \left[\int f(x)\,dx\right]\Big|_a^b = F(b) - F(a)$$

Consider R as





$V = \int_a^b \left(\text{Area of slice}\right) dx$



$\left(\int_c^d f(x,y)\, dy\right)$

area of slice

$$\iint_R f(x,y)\,dA = \int_a^b \left[\int_c^d f(x,y)\,dy\right] dx$$

or

$$\iint_R f(x,y)\,dA = \int_c^d \left[\int_a^b f(x,y)\,dx\right] dy$$

$\boxed{\text{Iterated Integrals}}$

Note: $f(x,y) = h(x)\,g(y)$

ex. $f(x,y) = \dfrac{x^2 \sin(y)}{\sqrt{1+x}} = \left(\dfrac{x^2}{\sqrt{1+x}}\right)(\sin y)$

then: $\iint_R f(x,y)\,dA = \left(\int_a^b h(x)\,dx\right)\left(\int_c^d g(y)\,dy\right)$

(ex) $f(x,y) = x^2 + xy + y^2$ over $\overset{x\text{'s}}{[0,2]} \times \overset{y\text{'s}}{[0,3]}$

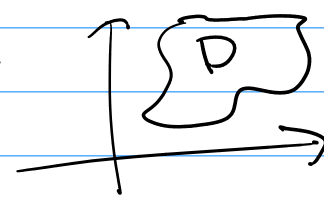
$$\iint_R (x^2+xy+y^2)\,dA = \int_0^2 \left[\int_0^3 (x^2+xy+y^2)\,dy\right] dx$$

$$= \int_0^2 \left[x^2 y + \tfrac{1}{2} x y^2 + \tfrac{1}{3} y^3 \Big|_0^3 \right] dx$$

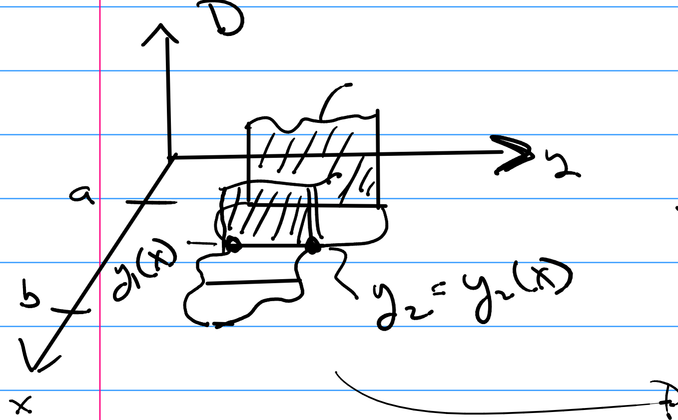
$$= \int_0^2 \left[3x^2 + \tfrac{9}{2} x + 9 \right] dx$$

$$= x^3 + \tfrac{9}{4} x^2 + 9x \Big|_0^2 = 8 + 9 + 18 - 0$$

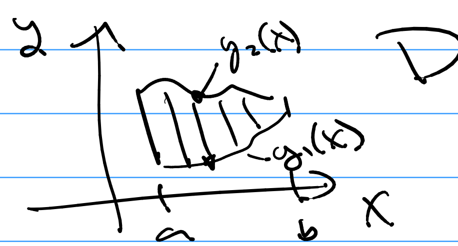
$$= \boxed{35} \text{ units}^3$$

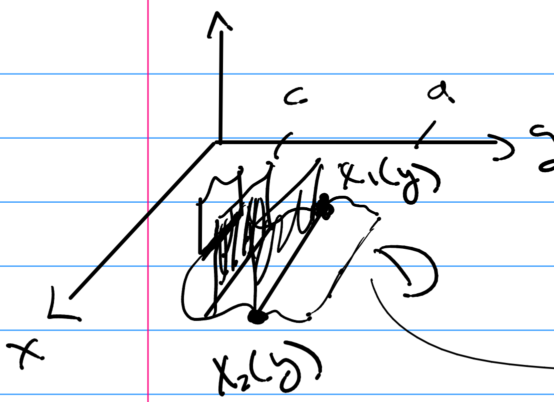
$\boxed{15.2}$ Double Integral over 

$$\iint f(x,y)\,dA$$

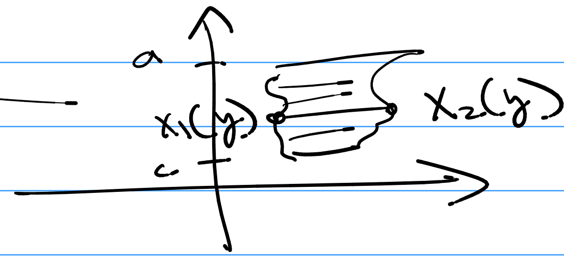


$$\int_a^b \left[\int_{y_1(x)}^{y_2(x)} f(x,y)\,dy \right] dx$$





$$\int_c^d \left[\int_{x_1(y)}^{x_2(y)} f(x,y)\,dx \right] dy$$



Properties

$D = D_1 \cup D_2 \cup D_3 \cup D_4$

$$\iint_D f\,dA = \iint_{D_1} f\,dA + \iint_{D_2} f\,dA + \iint_{D_3} f\,dA + \iint_{D_4} f\,dA$$

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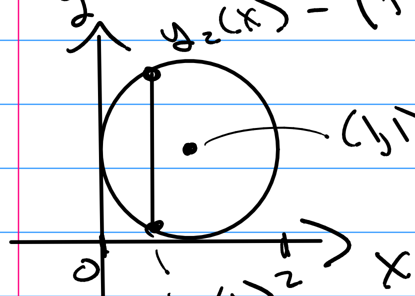
ex $\iint_D (x^2+y^2)\,dA$ D circle $(x-1)^2+(y-1)^2=1$

$(y-1)^2 = 1-(x-1)^2$

$y = 1 \pm \sqrt{1-(x-1)^2}$

$y_2(x) = 1+\sqrt{1-(x-1)^2}$



$y_1(x) = 1-\sqrt{1-(x-1)^2}$

$$\int_0^2 \left[\int_{1-\sqrt{1-(x-1)^2}}^{1+\sqrt{1-(x-1)^2}} (x^2+y^2)\,dy \right] dx = ?$$