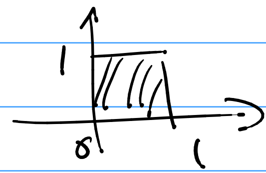
Math 344

~~Q's~~

$$\iint_R \sqrt{1+xe^{-y}}\,dA$$



Midpt

$n = \#$ of squares

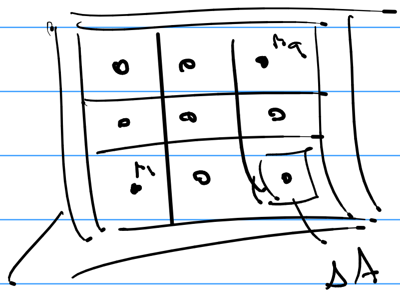
$n=1 = 1\cdot 1$

$n=4 = 2\cdot 2$

$n=16 = 4\cdot 4$

$n=64 = 8\cdot 8$

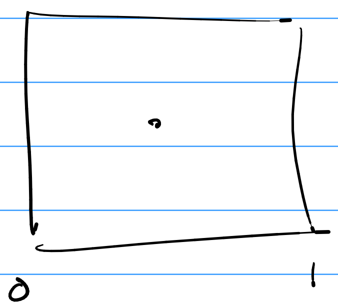
$$f(x,y) = \sqrt{1+xe^{-y}}$$



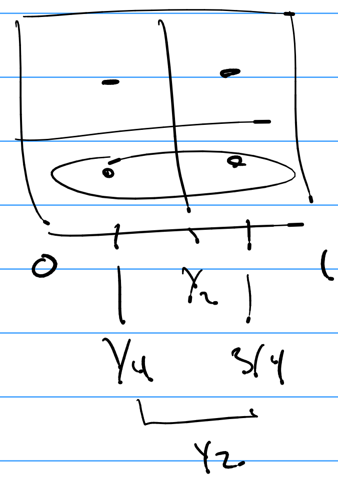
$$V \approx f(m_1)\Delta A + f(m_2)\Delta A + \dots + f(m_9)\Delta A$$

or

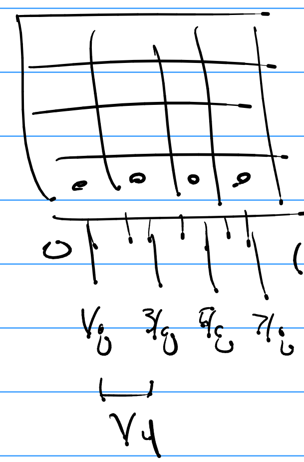
$$V \approx A\left[\frac{f(m_1)+f(m_2)+\dots+f(m_9)}{9}\right]$$



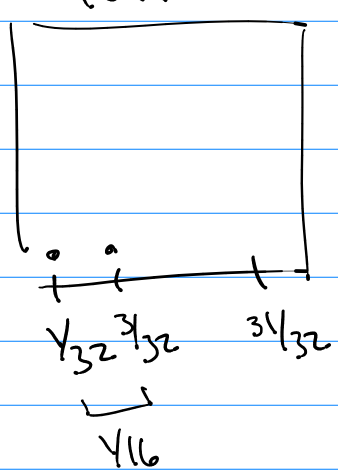
2x2



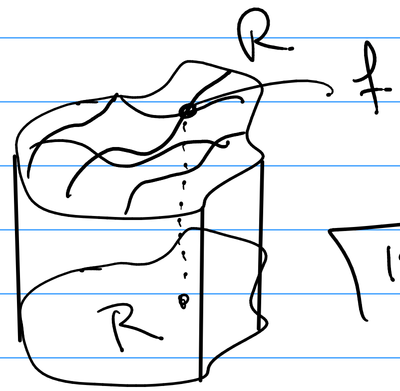
4 x 4



16x16

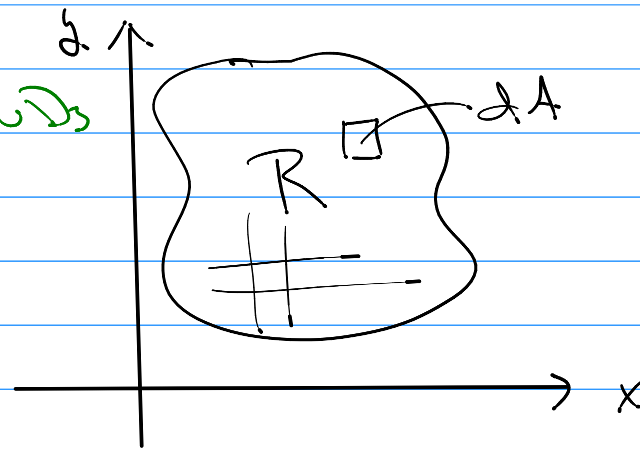


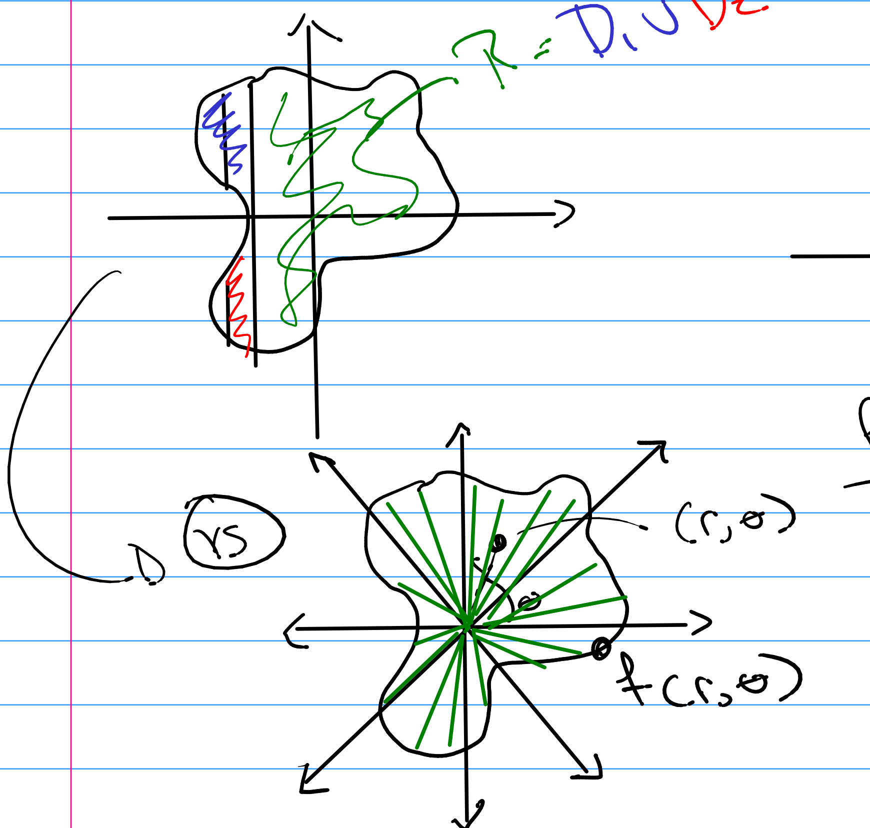
Volume $= \iint_R f \, dA$



$\boxed{\text{Is?}}$

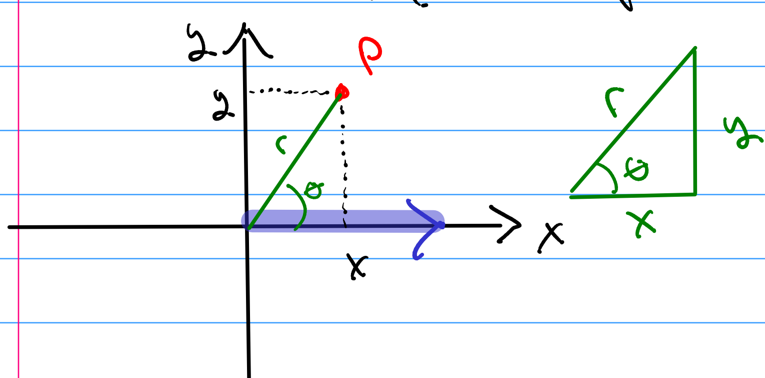
$$\iint_R f\,dA = \iint_R f \; \boxed{dx\,dy}$$





polar coordinates

Cartesian to/from polar



$\boxed{\text{Conversion eq'n's}}$

$x^2 + y^2 = r^2$

$y/x = \tan\theta$

$r\sin\theta = y$

$r\cos\theta = x$

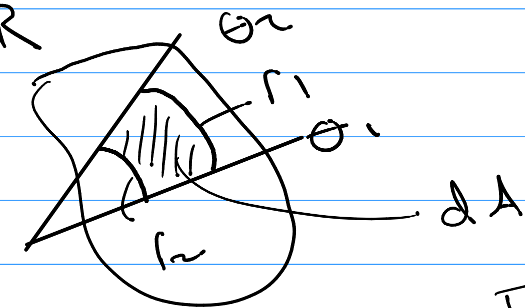
Goal

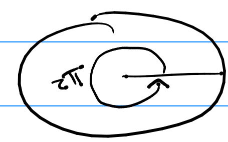
go from: $\iint_R \boxed{f}\, dA$ into an integral of r's θ's

Step 1 f = expression of r's and θ's

(use conversion eqn's if needed)

Step 2 $dA = ?$ $\quad r_1 \le r \le r_2 \quad \theta_1 \le \theta \le \theta_2$



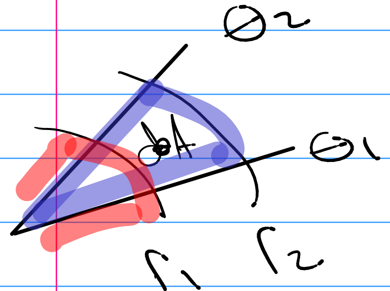


$A = \pi r^2$

know

$A = \boxed{\frac{1}{2}\theta r^2}$

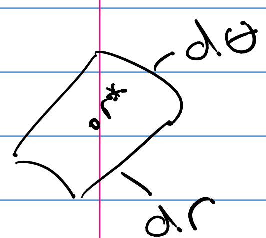
area of a θ sweep of a circle



$\boxed{\frac{1}{2}(\theta_2-\theta_1)\, r_2^2} - \boxed{\frac{1}{2}(\theta_2-\theta_1)\, r_1^2} = dA$

$\frac{1}{2}(\theta_2-\theta_1)\left[r_2^2 - r_1^2\right] = dA$

$\frac{1}{2}(\theta_2-\theta_1)(r_2+r_1)(r_2-r_1) = dA$



$\underbrace{\boxed{\frac{r_2+r_1}{2}}}_{= r^*}(r_2-r_1)(\theta_2-\theta_1) = dA$

So $dA = r\, dr\, d\theta$

So

use polar coord?

$$\text{Vol} = \iint_R f \, dA = \boxed{\iint_R f(r,\theta)\, r \, dr \, d\theta}$$

R defined in r, θ coord.

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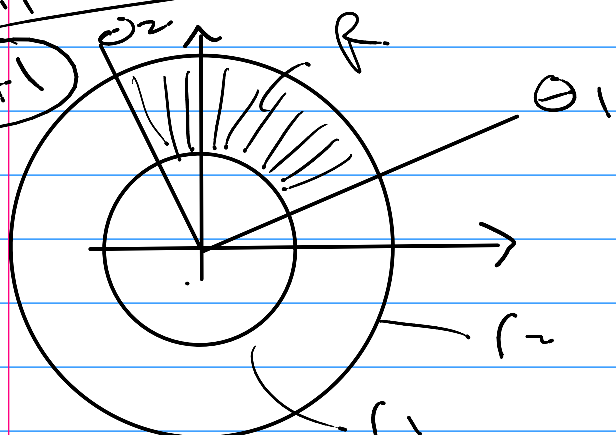
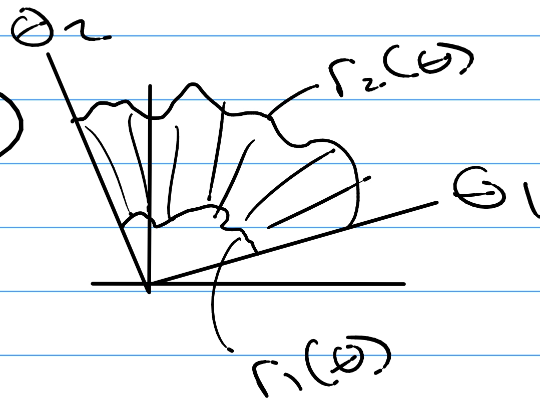
typical problems

#1



$$V = \int_{\theta_1}^{\theta_2} \left[\int_{r_1}^{r_2} f(r,\theta)\, r \, dr \right] d\theta$$

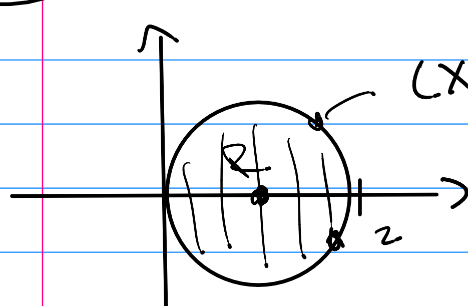
#2



$$V = \int_{\theta_1}^{\theta_2} \left[\int_{r_1(\theta)}^{r_2(\theta)} f(r,\theta)\, r \, dr \right] d\theta$$

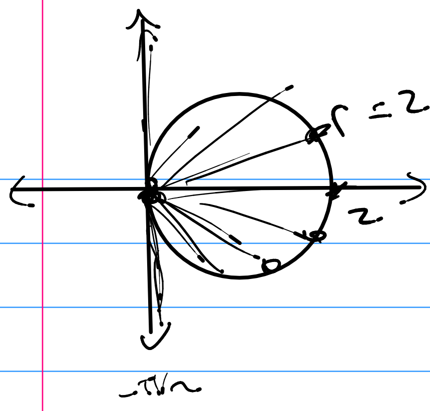
Ex $f(x,y) = x^2 + y^2$ $\qquad \iint_R f \, dA$

$(x-1)^2 + (y)^2 = 1^2$



in cartesian...

$$\int_0^2 \left[\int_{-\sqrt{1-(x-1)^2}}^{\sqrt{1-(x-1)^2}} x^2 + y^2 \, dy \right] dx$$



$r = 2\cos\theta \quad -\pi/2 \le \theta \le \pi/2$

$$\iint f \, dA$$

$$\int_{-\pi/2}^{\pi/2} \int_{0}^{2\cos\theta} r^3 \, dr \, d\theta$$