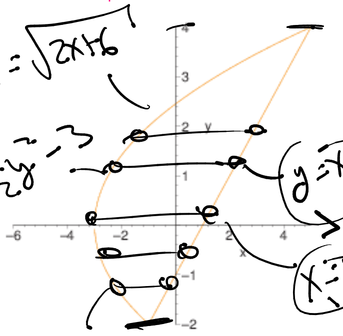
Math 344

$y = \sqrt{2x+6}$

$x = \frac{1}{2}y^2 - 3$



$y = -\sqrt{2x+6}$

Video Example

EXAMPLE 3 Evaluate $\iint_D xy\, dA$, where D is the region bounded by the line $y = x - 1$ and the parabola $y^2 = 2x + 6$. $\longrightarrow$ $x = \frac{1}{2}y^2 - 3$

$y = x-1$ $\quad$ $x = y+1$

SOLUTION The region D is shown in the figure. Again D is both type I and type II, but the description of D as a type I region is more complicated because the lower boundary consists of two parts. Therefore we prefer to express D as a type II region:

$$D = \left\{(x, y) \,\middle|\, -2 \le y \le 4,\ \tfrac{1}{2}y^2 - 3 \le x \le y + 1\right\}.$$

$$\int_{-2}^{4} \left[\int_{\frac{1}{2}y^2-3}^{y+1} xy\, dx \right] dy$$

Then this equation gives

$$\iint_D xy\, dA = \int_{-2}^{4}\int_{1/2y^2-3}^{y+1} xy\, dx\, dy$$

$$= \int_{-2}^{4} \left[\boxed{} \right]_{x = 1/2y^2 - 3}^{y+1} dy$$

$$= \frac{1}{2}\int_{-2}^{4} y\left[(y+1)^2 - \left(\boxed{\tfrac{1}{2}y^2 - 3}\right)^2\right] dy$$

$((1/2)*y^2) - 3$

$$= \frac{1}{2}\int_{-2}^{4}\left(\boxed{} + 4y^3 + 2y^2 - 8y\right) dy$$

$$y\int_{\frac{1}{2}y^2-3}^{y+1} x\, dx = \boxed{\tfrac{1}{2}yx^2}\Bigg|_{x = \frac{1}{2}y^2-3}^{x = y+1}$$

$$= \frac{1}{2}y\left[(y+1)^2 - \left(\tfrac{1}{2}y^2 - 3\right)^2\right]$$

Find the volume of the given solid.

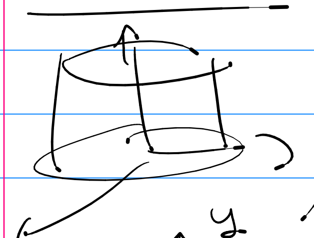
Bounded by the cylinders $x^2 + y^2 = 16r^2$, $y^2 + z^2 = 16r^2$

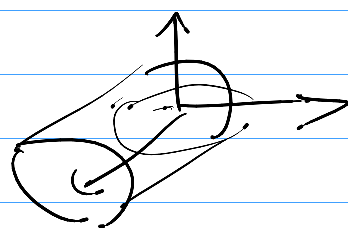
3D

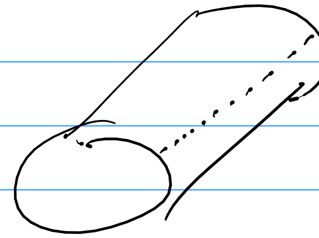
$z = \sqrt{(4r)^2 - y^2}$

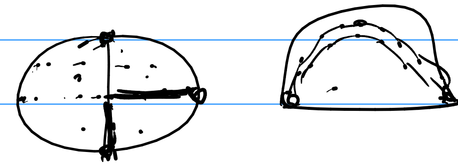
xy plane

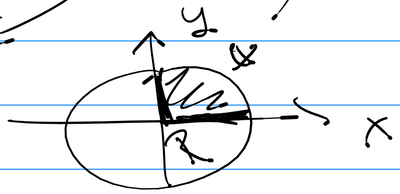
$x^2 + y^2 = (4r)^2$ $\qquad$ $y^2 + z^2 = (4r)^2$







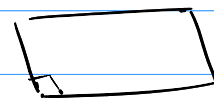


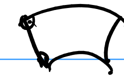


$$\iint_R \sqrt{(4r)^2 - y^2}\, dA$$

$$\boxed{\int_c^d \int_a^b}\; d\square\, d\triangle$$

↳ constants : "rectangular" region

$dx\,dy$ → "rectangular"  $dx\,dy$ area

$dr\,d\theta$ → "rectangular"  $r\,dr\,d\theta$ area

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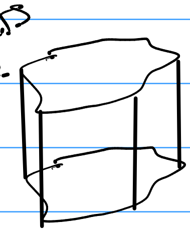
up to now ... $\boxed{\iint_R f\,dA}$ (double integrals) can be done.

15.4 Applications

① Volume

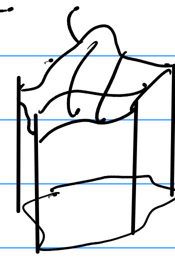
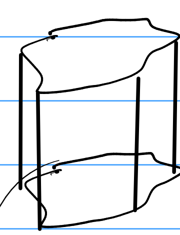
② Area

$\iint_R (1)\,dA =$ this objects volume  1

b/c cylinder = $1 \cdot$ area(base)

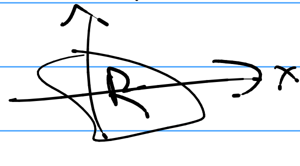
So $\iint_R (1)\,dA$ = # for area of base (units wrong)

③ Ave. height $z = f(x,y)$

$\iint_R f(x,y)\,dA = V$ Vol   $\bar{f}$ = const

f average 

want: V = Area of base $\cdot$ $\boxed{\text{height}}$



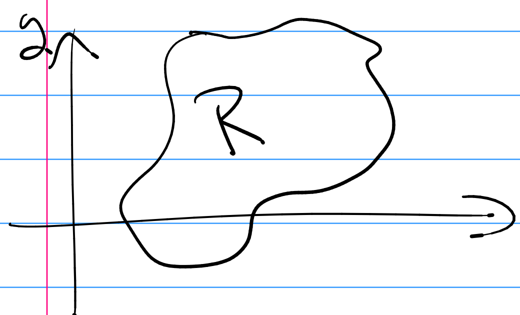
$$f_{ave} = \bar{f} = \frac{1}{A}\boxed{\iint_R f(x,y)\,dA}$$

where A = area of region R.

Physics:

① Density = $\left(\frac{\text{amount of "stuff"}}{\text{unit area}}\right)$

a) Mass density = $\rho = \frac{\text{Mass}}{\text{area}}$



$\rho(x,y) = \frac{\text{Mass of material}}{\text{area}}$ @ (x,y) position

$$\boxed{\text{Mass} = \iint_R \rho(x,y)\,dA}$$

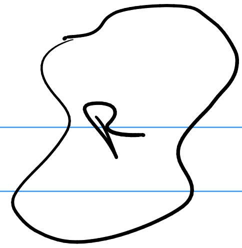
ex

$$\rho(x^*,y^*)$$

$\rho(x,y) = \frac{\text{people}}{\text{area}}$

b) charge Density $\sigma(x,y) = \frac{\text{charge}}{\text{area}}$

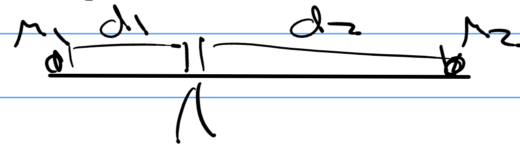
total charge $Q = \iint_R \sigma(x,y)\,dA$

Sheet 

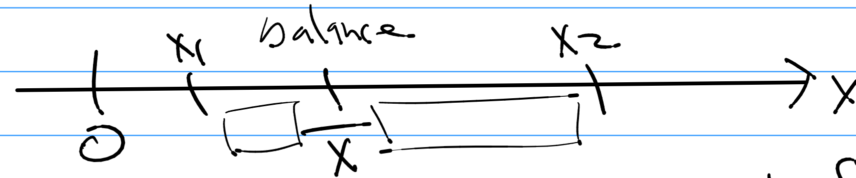
with density $\rho(x,y)$

→ center of mass?

law of levers:



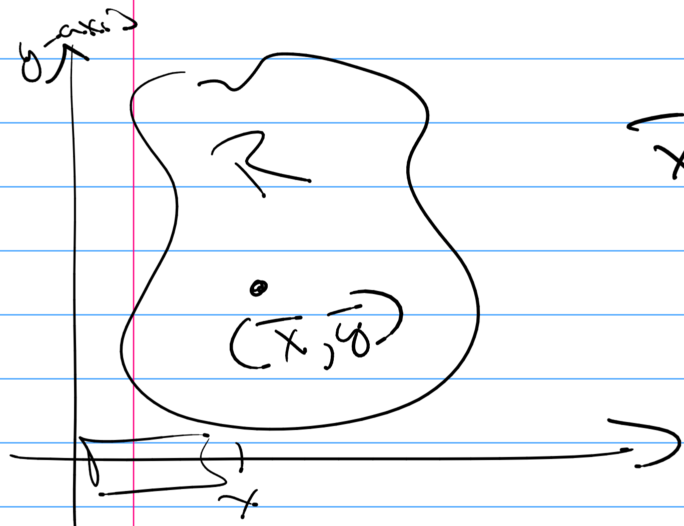
$m_1 d_1 = m_2 d_2$



$$\bar{x} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$$

moment of m_2

$$\bar{x} = \frac{\text{sum of moments according to x-coord.}}{\text{total mass}}$$



$$\bar{x} = \frac{\iint_R x\,\rho(x,y)\,dA}{M} \qquad = M_{y\text{-axis}}$$

$$\bar{y} = \frac{\iint_R y\,\rho(x,y)\,dA}{M} \qquad = M_{x\text{-axis}}$$

$$M = \iint_R \rho(x,y)\,dA$$