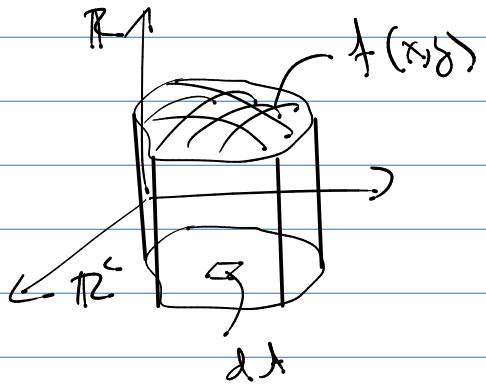
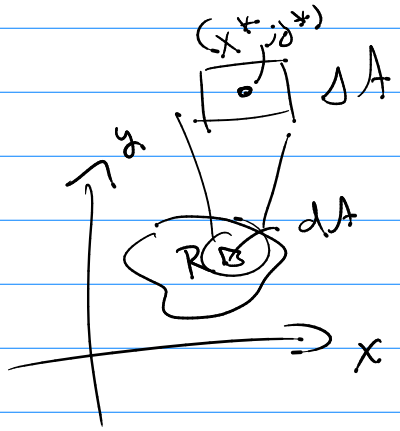
Math 344

Ch 14 $f: \mathbb{R}^n \to \mathbb{R}$ f_x, ∇f, etc

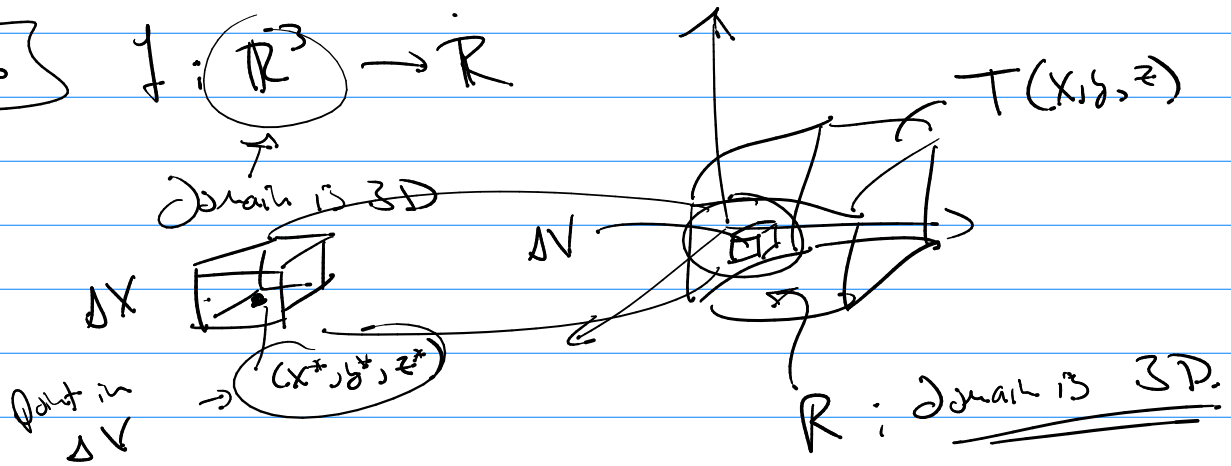
Ch 15.1 – 15.5 $f: \mathbb{R}^2 \to \mathbb{R}$



$$\iint_R f \, dA$$



15.6 $f: \mathbb{R}^3 \to \mathbb{R}$



$$f(x^*, y^*, z^*)\,\Delta V$$

4D cylinder has a 4D Volume.
↳ 4D Content.

→ add all these 4D cylinders across the domain.

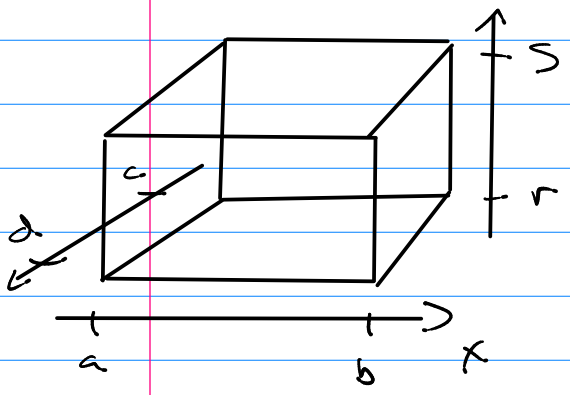
$$\lim_{r,m,n \to \infty} \sum_{i=1}^{r} \sum_{j=1}^{m} \sum_{k=1}^{n} f(x^*_{ijk}, y^*_{ijk}, z^*_{ijk})\,\Delta V$$

Def $\iiint_R f\,dV = \lim_{l,m,n\to\infty} \sum_i \sum_j \sum_k f(x^*_{ijk}, y^*_{ijk}, z^*_{ijk})\,\Delta V$

R → 3D region → find content of 4D object

Fubini's th^n region R is a rectangular region

$a \le x \le b,\ c \le y \le d,\ r \le z \le s$



$$\iiint_R f\,dV = \int_a^b \left[\int_c^d \left[\int_r^s f\,dz\right] dy\right] dx$$

Ex find Volume of R $\iiint_R (1)\,dV$

Vol. of rectangular region $\underbrace{[-1,3]}_{x} \times \underbrace{[2,5]}_{y} \times \underbrace{[0,2]}_{z}$

$$\int_{-1}^{3}\int_{2}^{5}\left[\int_{0}^{2}(1)\,dz\right] dy\,dx$$

$z\big|_0^2$

$$\int_{-1}^{3}\int_{2}^{5}(2)\,dy\,dx = \int_{-1}^{3} 6\,dx$$

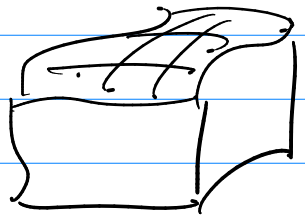
$2y\big|_2^5$

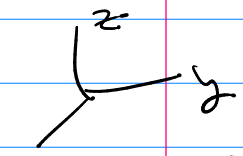
$$= 6x\Big|_{-1}^{3}$$

$$= 18 + 6 = \boxed{24}$$

General regions

$$\iiint_R f\,dV$$





$dz \; dA$

$dx \; dA$

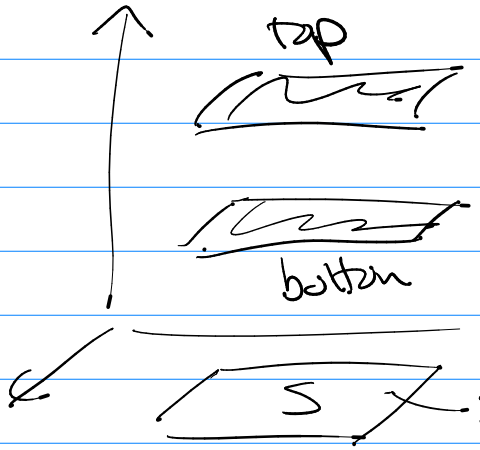
$dy \; dA$

3 ways:

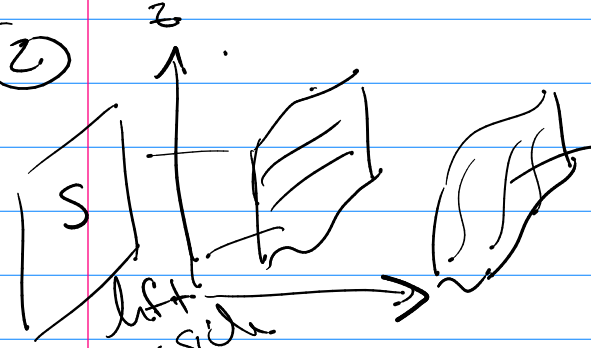
(1) $\iiint_R f\,dV = \iint_S \left[\int_{z\text{-lower}}^{z\text{-upper}} f\,dz \right] dA$

z-upper: top of R

z-lower: bottom of R



(2)



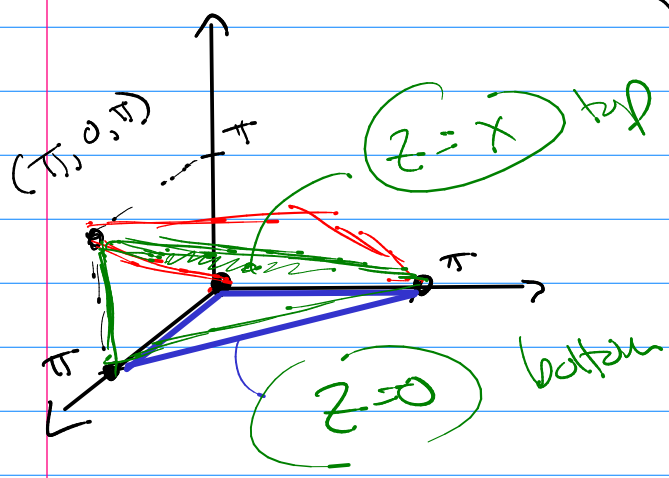
$$\iint_S \left[\int_{\text{left side}}^{\text{right side}} f\,dy \right] dA$$

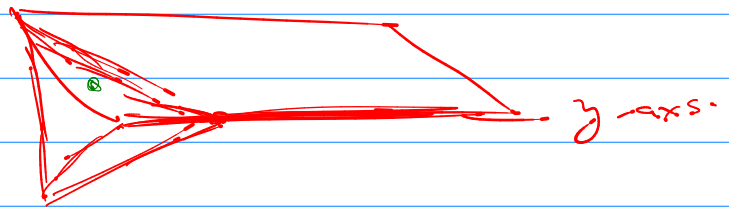
$y = f(x,z)$

(3) $$\iint_S \left[\int_{\text{back}}^{\text{front}} f\,dx \right] dA$$

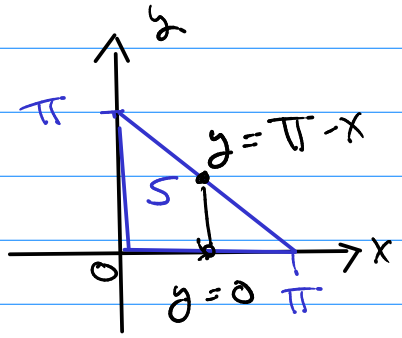
$x = f(y,z)$

Ex) $\iiint_R (\sin y)\, dV$ R is volume below $z = x$ above region of a triangle with vertices $(0,0,0)$, $(\pi,0,0)$, $(0,\pi,0)$





$$\iint_S \left[\int_{z=0}^{z=x} \sin(y)\, dz \right] dA$$



$$\int_0^{\pi} \left[\int_0^{\pi - x} \left[\int_0^{x} \sin(y)\, dz \right] dy \right] dx$$

Apps $\quad \rho(x,y,z) = \frac{\text{mass}}{\text{vol}}$ density

$$M = \iiint_E \rho\, dV \quad \text{total mass}$$

$$\bar{x} = \frac{1}{M} \iiint_E x\rho\, dV \qquad \bar{y} = \frac{1}{M} \iiint_E y\rho\, dV \qquad \bar{z} = \frac{1}{M} \iiint_E z\rho\, dV$$