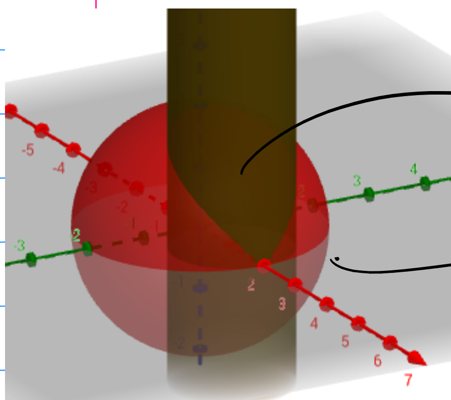
Math 344

Q's 15.5 #11 $x^2+y^2+z^2=a^2$, $x^2+y^2=ax$

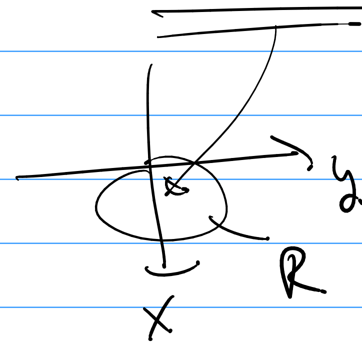
$x^2-ax+(\frac{a}{2})^2+y^2=0+(\frac{a}{2})^2$



z_2 = sphere

z_1 = xy plane

$(x-a/2)^2+y^2=(a/2)^2$



$$SA=\iint_R \sqrt{1+(f_x)^2+(f_y)^2}\,dA$$

f is sphere (top)

$$z=f(x,y)=(a^2-x^2-y^2)^{1/2}$$

$$f_x=\frac{-x}{\sqrt{a^2-x^2-y^2}} \qquad f_y=\frac{-y}{\sqrt{a^2-x^2-y^2}}$$

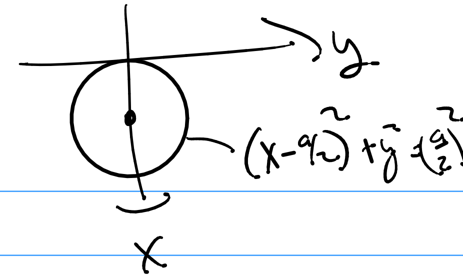
$$SA=\iint_R \left(1+\frac{x^2}{a^2-x^2-y^2}+\frac{y^2}{a^2-x^2-y^2}\right)^{1/2} dA$$

$$SA=\iint_R \left(\frac{a^2-x^2-y^2+x^2+y^2}{a^2-x^2-y^2}\right)^{1/2} dA=\iint_R \left(\frac{a^2}{a^2-x^2-y^2}\right)^{1/2} dA$$

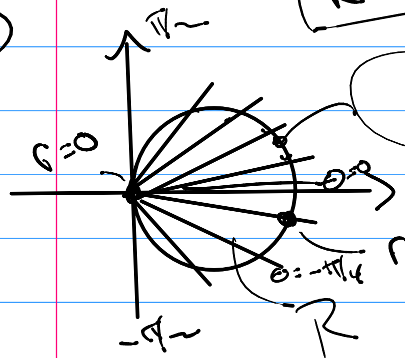
Note: $\sqrt{a^2}=|a|=a$ b/c a is pos (radius of sphere)

$$SA = a \boxed{\iint_R \frac{1}{\sqrt{a^2-x^2-y^2}}\, dA}$$

R^2_o

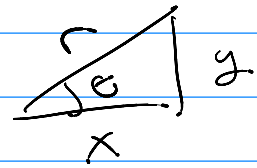


Gr



$x^2+y^2=ax$

a



$r = a\cos\theta$

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(a) $$\frac{1}{\sqrt{a^2-x^2-y^2}} = \frac{1}{\sqrt{a^2-r^2}}$$

(b) $$SA = a\int_{-\pi/2}^{\pi/2}\left[\int_0^{a\cos\theta} \frac{1}{\sqrt{a^2-r^2}}\, r\,dr\right]d\theta = -\frac{a}{2}\int_{-\pi/2}^{\pi/2}\left[\int_{a^2}^{a^2\sin^2\theta} u^{-1/2}\,du\right]d\theta$$

sub:

let $u = a^2 - r^2 \quad du = -2r\,dr$

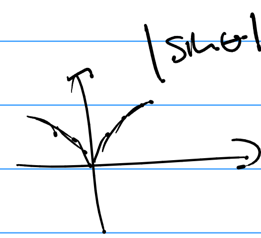
@ $r=0 \quad u = a^2$

@ $r = a\cos\theta \quad u = a^2 - a^2\cos^2\theta = a^2(1-\cos^2\theta) = a^2\sin^2\theta$

$$SA = -a\int_{-\pi/2}^{\pi/2}\left[u^{1/2}\Big|_{a^2}^{a^2\sin^2\theta}\right]d\theta$$

$$SA = -a\int_{-\pi/2}^{\pi/2}\left(\sqrt{a^2\sin^2\theta} - \sqrt{a^2}\right)d\theta$$

$$SA = -a^2\int_{-\pi/2}^{\pi/2}\left(|\sin\theta| - 1\right)d\theta$$



$$|x| = \begin{cases} x & \text{if } x \ge 0 \\ -x & \text{if } x < 0 \end{cases}$$

$$SA = -a^2\left[\int_{-\pi/2}^{0} -\sin\theta \, d\theta + \int_{0}^{\pi/2} \sin\theta \, d\theta - \int_{-\pi/2}^{\pi/2} 1 \, d\theta\right]$$

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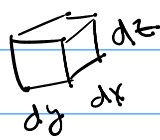
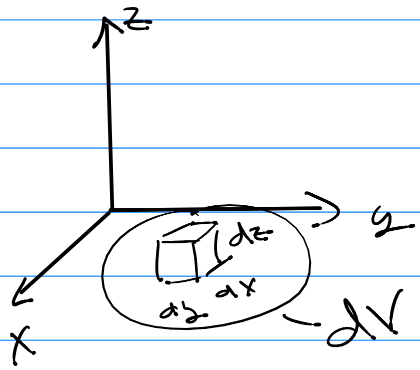
triple integrals

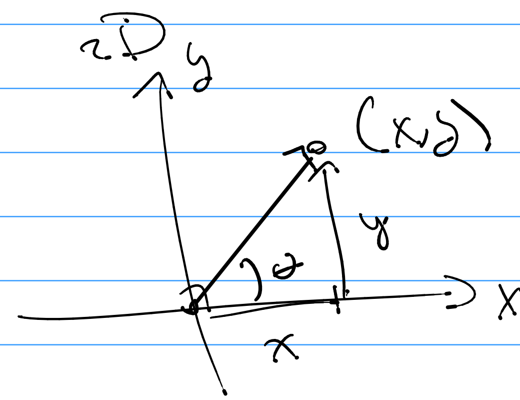
$$\iiint_R f \, \boxed{dV}$$

$dV \rightarrow$ 

$dV = dx\,dy\,dz$

Cartesian:

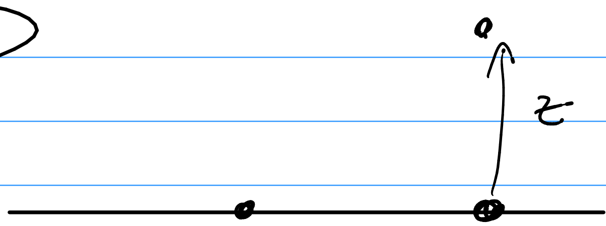




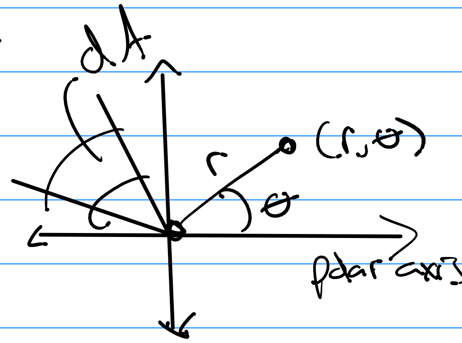
(r,θ) vs (x,y)

$\boxed{\text{other coord. systems for 3D}}$

(A)



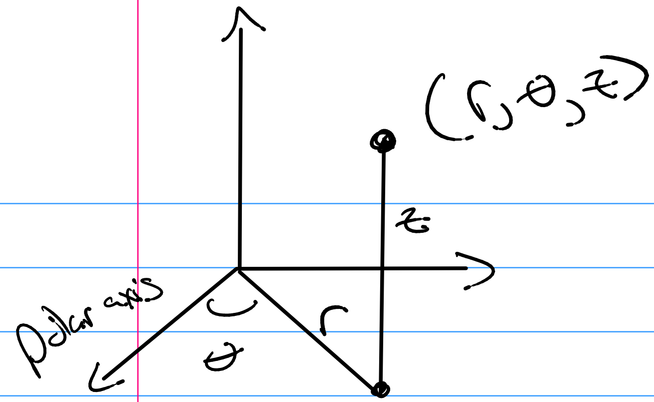
Vertical.

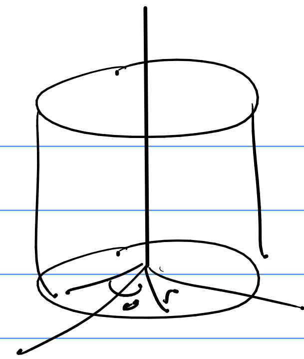


plane

together?

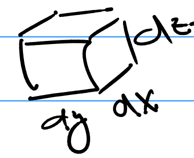
$\boxed{\text{cylindrical coord.}}$



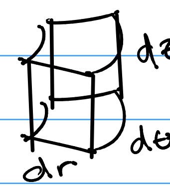


$$\iiint_R f \, dV$$

cartesian



cylindrical



$dV = \left(\begin{smallmatrix}\text{area}\\ \text{base}\end{smallmatrix}\right)$ height

$dV = r\,dr\,d\theta\,dz$

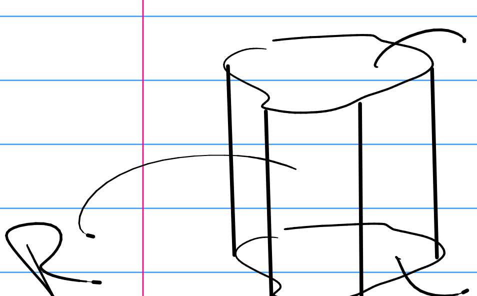
cartesian: $\iiint_R f \, dx\,dy\,dz$

only have x, y, z's

cylindrical: $\iiint_R f \; r\,dr\,d\theta\,dz$

only have r, θ, z's

we use cylindrical coord when object "looks" cylindrical with base represented best in polar form.



top: $z_2 = f_2(r,\theta)$

bottom $z_1 = f_1(r,\theta)$

$$\iiint_R f \; r\,dr\,d\theta\,dz = \iint_{\boxed{\text{base}}} \left[\int_{f_1(r,\theta)}^{f_2(r,\theta)} f(z,r,\theta)\, dz \right] r\,dr\,d\theta$$