Math 344

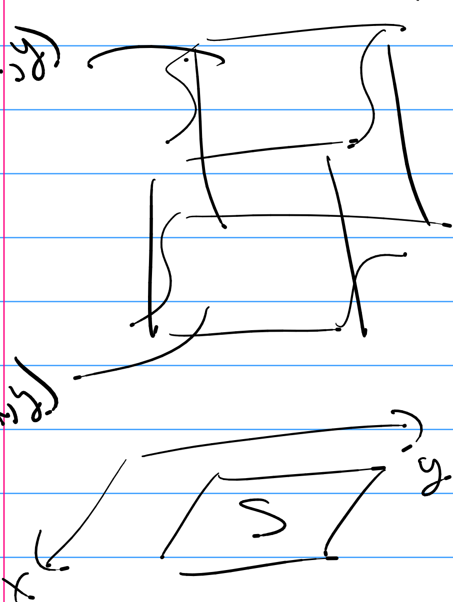
Triple Integrals $\iiint_R f\,dV$

① Cartesian $dV = dx\,dy\,dz \rightarrow \iiint_R f\,dx\,dy\,dz$

! only have x's, y's, z's

(ex) Natural top, bottom

top $z = f_2(x,y)$



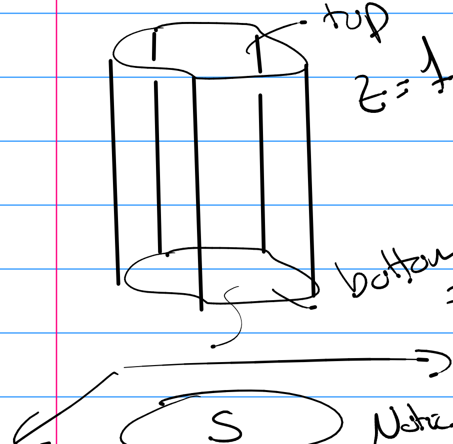
$z = f_1(x,y)$

$$\iiint_R \Big[\overbrace{5x + 2y + \cos z}^{f(x,y,z)}\Big]\,dV$$

$$\iint_S \left[\int_{f_1(x,y)}^{f_2(x,y)} (5x + 2y + \cos z)\,dz\right] dA$$

② **cylindrical** $\iiint_R f\,dV$

top $z = f_2(r,\theta)$



bottom $z = f_1(r,\theta)$

$$\iiint_R f\; r\,dr\,d\theta\,dz$$

! only see r, θ, z

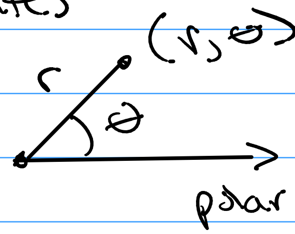
Notice polar coord would work well here

so far cylindrical $\iint_S \left[\int_{f_1(r,\theta)}^{f_2(r,\theta)} f(r,\theta,z)\, dz \right] r\, dr\, d\theta$

15.8 Spherical Coordinates

1st Polar Coordinates

2D

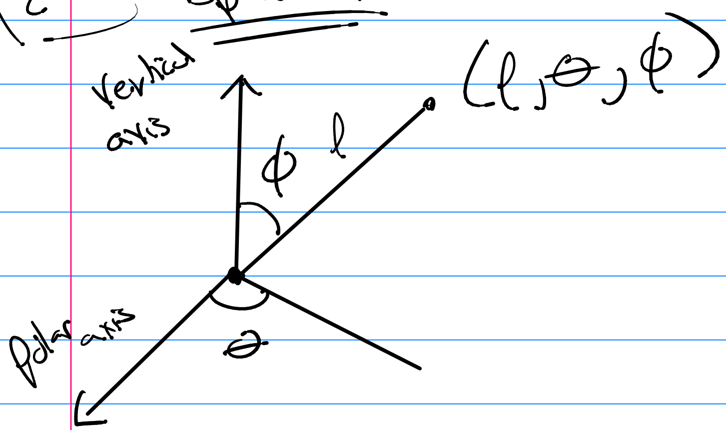


can restrict the coord.

ex $r \geq 0$

$0 \leq \theta < 2\pi$

2nd Spherical



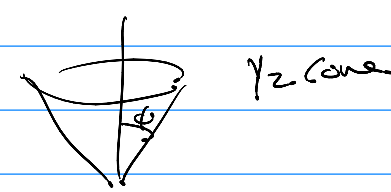
restrictions:

$$\boxed{\begin{gathered}\rho \geq 0 \\ 0 \leq \phi \leq \pi\end{gathered}}$$

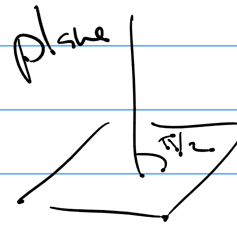
shapes: $\rho = \text{const}$ sphere centered @ origin

$\theta = \text{const}$

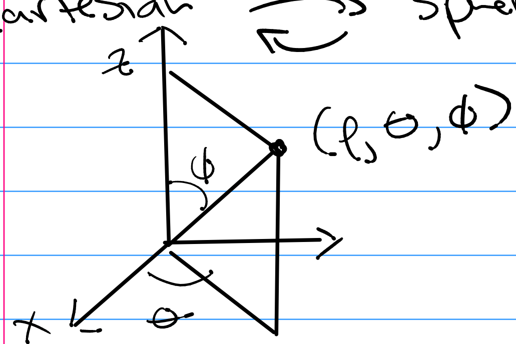
$\phi = \text{const}$

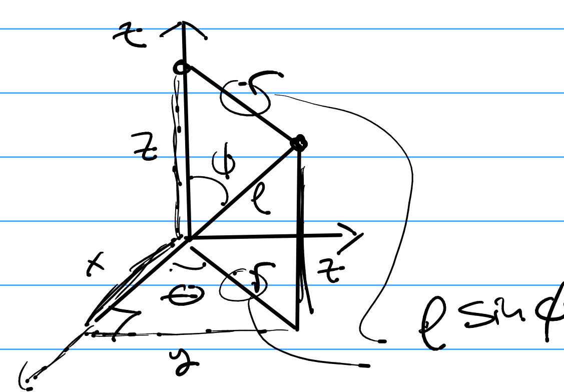


but $\phi = \pi/2 \to$ plane

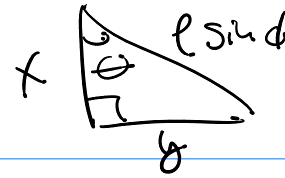


Cartesian $\rightleftarrows$ spherical





$$\boxed{\begin{aligned} z &= \rho\cos\phi \\ y &= \rho\sin\phi\,\sin\theta \\ x &= \rho\sin\phi\,\cos\theta \\ x^2+y^2+z^2 &= \rho^2 \end{aligned}}$$



---

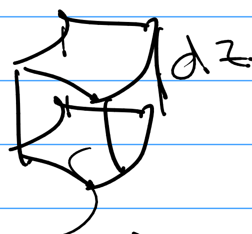
triple integral

$$\iiint_R f \, dV \leftarrow ?$$

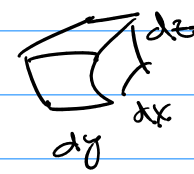
need only ρ, θ, ϕ

spherical wedge

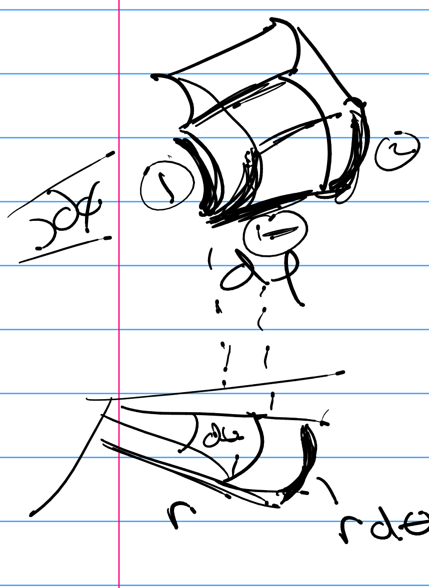
Cylindrical



Cartesian



$dV = dx\,dy\,dz$



$dV = (r\,dr\,d\theta)\,dz$

① $\rho\, d\phi$

② $(\rho\sin\phi)\, d\theta$

$$dV = (\rho\, d\phi)(\rho\sin\phi\, d\theta)\, d\rho$$

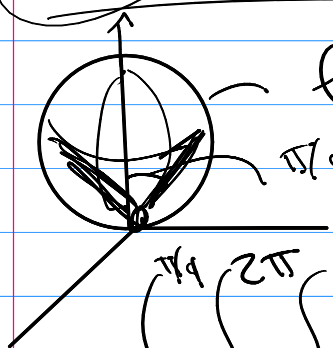
$$dV = \boxed{\rho^2 \sin\phi\, d\rho\, d\theta\, d\phi}$$

So $\iiint_R f\,dV = \iiint_R (f)(\rho^2 \sin\phi)\,\boxed{d\rho}\,d\theta\,d\phi$

put into ρ, θ, ϕ variables.

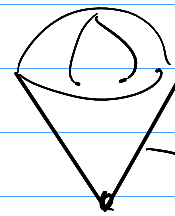
$\iiint_R (1)\,dV$

Volume of ice cream cone.



$\rho = \cos\phi$

$\pi/4$



$\phi = \pi/4$

$$\int_0^{\pi/4}\int_0^{2\pi}\int_0^{\cos\phi} (1)\,\rho^2 \sin\phi \, d\rho\, d\theta\, d\phi = ?_0 \text{ finish}$$