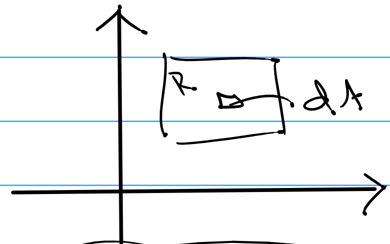
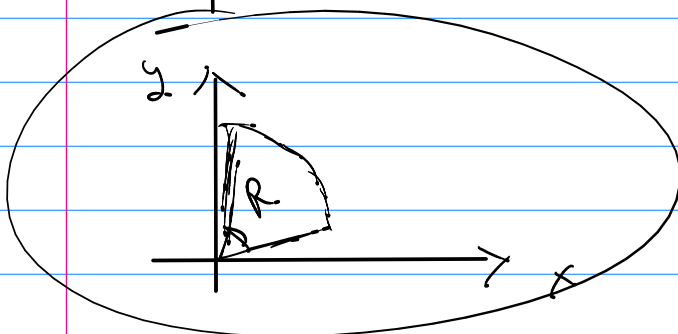
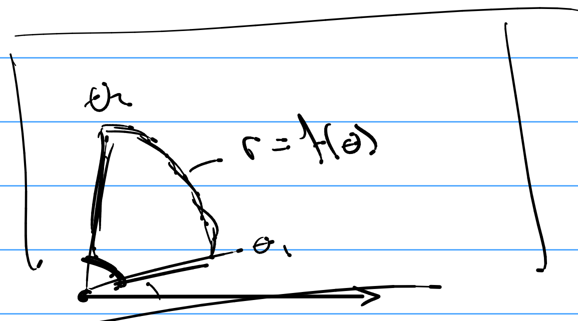
Math 344

$$\iint_R f(x,y)\,dA = \iint_R f(x,y)\,dx\,dy$$



rectangular

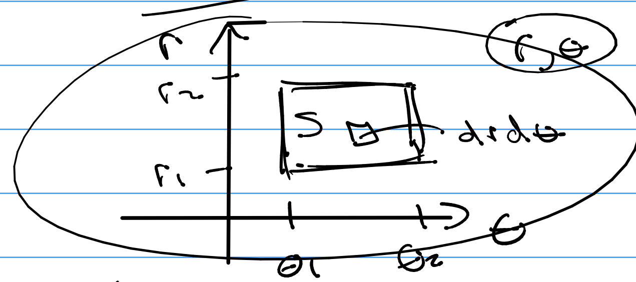




$$\iint_{Q} f(x,y)\,\boxed{dx\,dy}$$

x,y

$$dA = r\,dr\,d\theta$$

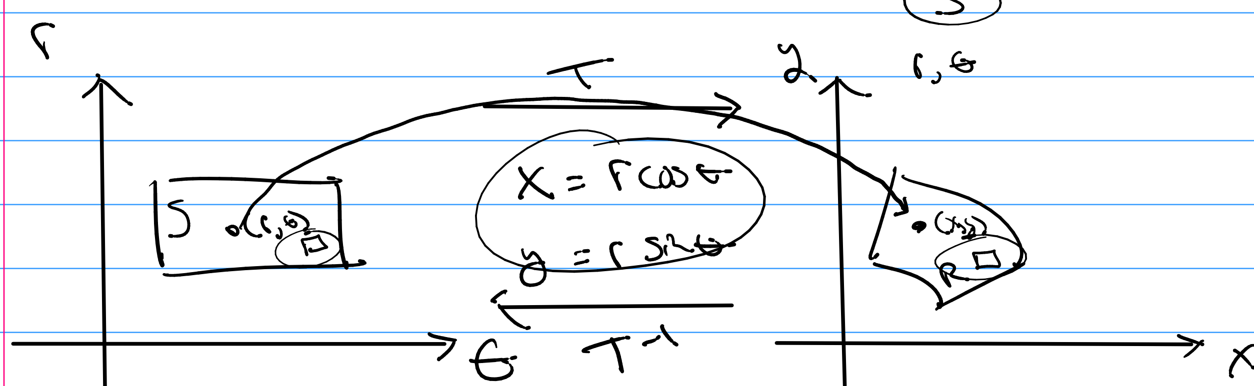


$x = r\cos\theta \rightarrow x = x(r,\theta)$

$y = r\sin\theta \qquad y = y(r,\theta)$

$$\iint_S f(r\cos\theta, r\sin\theta)\,\boxed{r}\,\underline{dr\,d\theta}$$

dA



$$\underbrace{\begin{cases} x = r\cos\theta = x(r,\theta) \\ y = r\sin\theta = y(r,\theta) \end{cases}}_{T}$$

$$\underbrace{\begin{cases} r = (x^2+y^2)^{1/2} = r(x,y) \\ \theta = \tan^{-1}\left(\dfrac{y}{x}\right) = \theta(x,y) \end{cases}}_{T^{-1}}$$

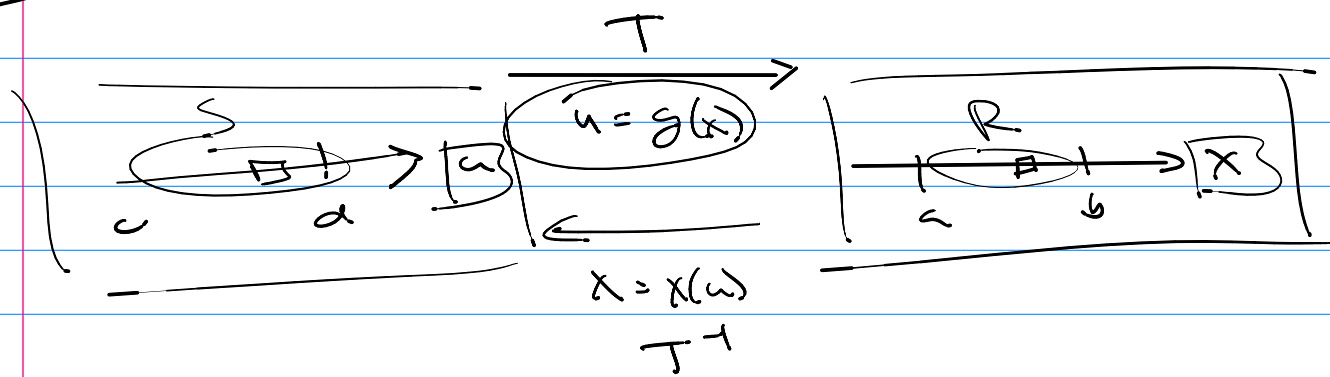
Substitution $\displaystyle\int_a^b f(x)\,dx$

$$\int_a^b \boxed{\overbrace{f(g(x))\,g'(x)}^{f(x)}}\,dx = \int_c^d f(u)\,du$$

let $u = g(x)$ $\qquad c = g(a)$

$du = g'(x)\,dx$ $\qquad d = g(b)$

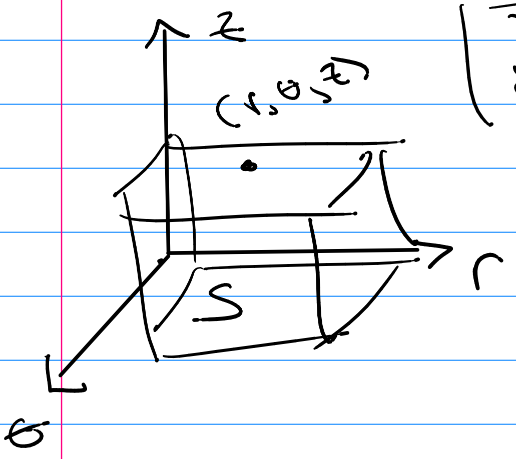
1D



3D

Cartesian vs cylindrical

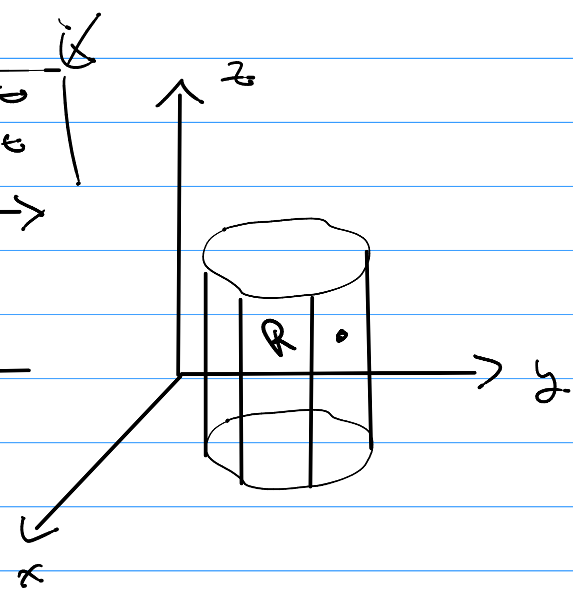
(x, y, z) $\qquad (r, \theta, z)$



$$x(r,\theta,z) = r\cos\theta$$
$$y(r,\theta,z) = r\sin\theta$$
$$z(r,\theta,z) = z$$

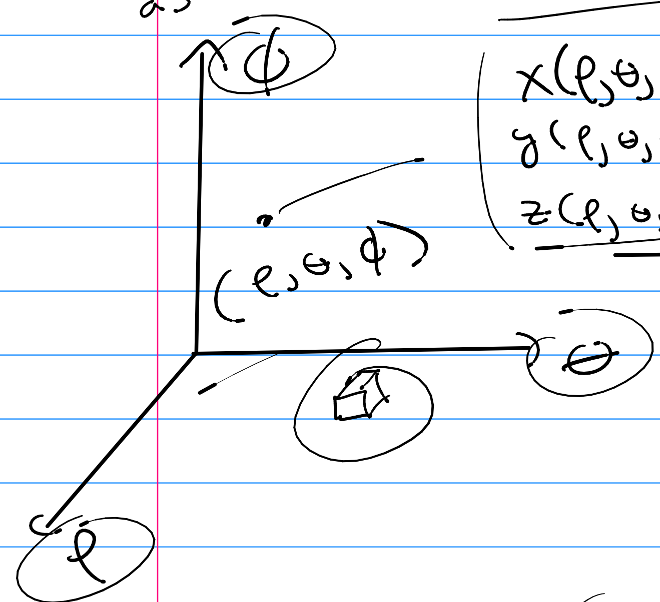
T

T^{-1}

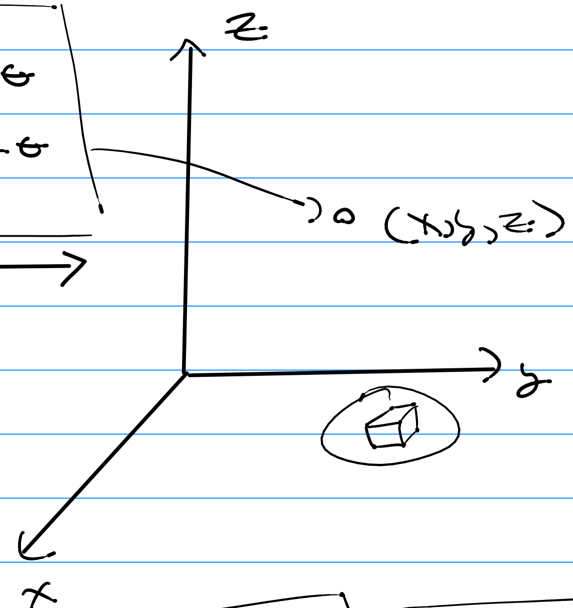


$$\iiint_R f(x,y,z)\,dV = \iiint_S f(r,\theta,z)\,r\,dr\,d\theta\,dz$$

do the same as above for spherical...



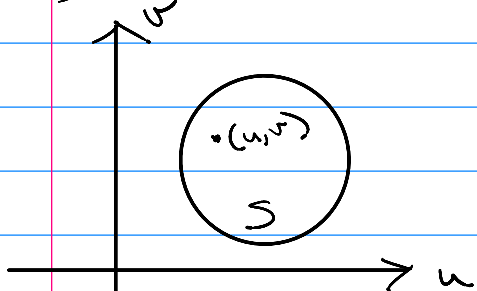
$$x(\rho,\theta,\phi) = \rho\sin\phi\cos\theta$$
$$y(\rho,\theta,\phi) = \rho\sin\phi\sin\theta$$
$$z(\rho,\theta,\phi) = \rho\cos\phi$$
$$\xrightarrow{\quad T \quad}$$



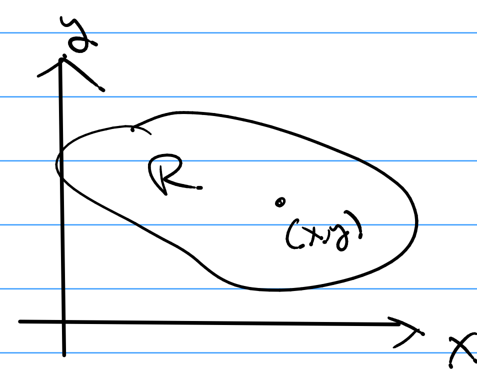
$$\iiint_R f(x,y,z)\,\underbrace{dV}_{dx\,dy\,dz} = \iiint_S f(\rho,\theta,\phi)\,\boxed{\rho^2\sin\phi}\,\boxed{d\rho\,d\theta\,d\phi}$$

2D

generalize this



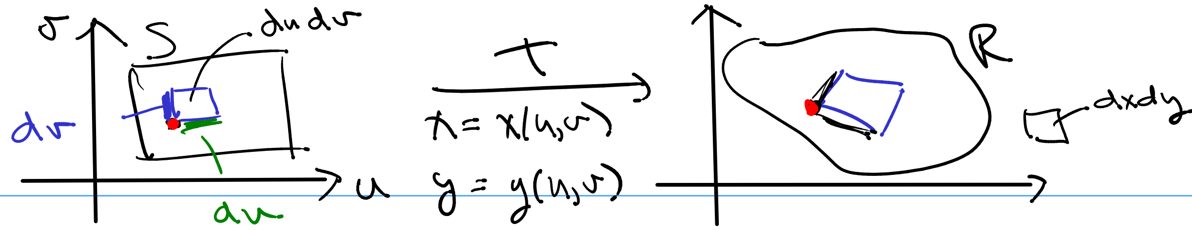
$$x = x(u,v)$$
$$y = y(u,v)$$
$$\xrightarrow{\quad T \quad}$$



Note: normally T (our transform) will be restricted by making $x(u,v)$, $y(u,v)$ have continuous 1st order partials. (C^1 transforms)

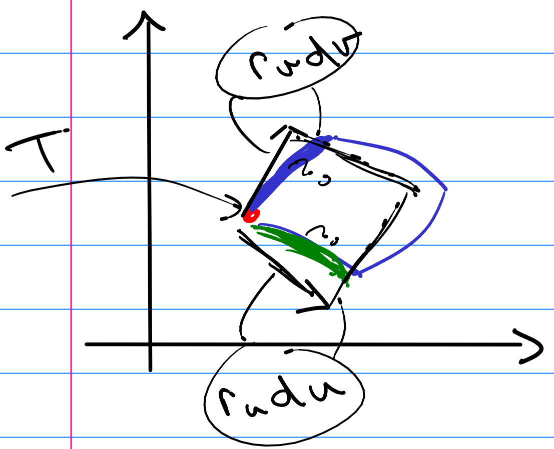
Now: $$\underbrace{\iint_R f(x,y)\,dA}_{x,y \text{ coord.}} \overset{?}{=} \iint_S f(x(u,v),y(u,v))\,\boxed{?}$$

small area part?



T in vector form $\vec{r} = \langle x(u,v), y(u,v) \rangle$

$\vec{r}_u = \langle x_u, y_u \rangle$ $\vec{r}_v = \langle x_v, y_v \rangle$



small area ≈ area of parallelogram

$\rightarrow |r_v dv \times r_u du|$

$= |r_u \times r_v| \, du \, dv$

$r_u = \langle x_u, y_u, 0 \rangle$ $r_v = \langle x_v, y_v, 0 \rangle$

$$r_u \times r_v = \begin{vmatrix} i & j & k \\ x_u & y_u & 0 \\ x_v & y_v & 0 \end{vmatrix} = \text{det?}$$